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Enhancing bridge performance following earthquakes using Markov decision process

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ABSTRACT

Bridges are an important type of transportation infrastructures. Once more than one bridge in a transportation network is extensively or completely damaged in a severe earthquake, the traffic capacity of the network may degrade significantly, resulting in considerable social and economic consequences to the community served. The principles of build back better (BBB) require that the performance level of a critical infrastructure facility should be properly determined in the post-hazard recovery process to reduce the future risk. This paper proposes a continuous-time Markov decision process framework based on life-cycle cost (LCC) for determining the optimum earthquake-resistant levels of the rebuilt bridges. Compared to other optimisation methodologies, the proposed one owns two distinctive features: (1) network-level LCC analyses are performed in the context of BBB principles; (2) optimum decisions are made sequentially at random time points of earthquake occurrence, and the optimum earthquake-resistant level of a rebuilt bridge depends on the performance levels of other bridges in the network. A hypothetical transportation network is investigated to demonstrate the application of the proposed methodology. In particular, the sensitivity of the optimum policy on network topology is studied.

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1. Introduction

A transportation network consists of roads, bridges, and tunnels, which are spatially distributed but connected to each other to meet the needs of the community that they serve. Among these infrastructural facilities, bridges have been proved to be vulnerable to seismic hazard by past earthquake events (Basoz & Kiremidjian, 1998, Han et al., 2009). Severe damages of bridges in an earthquake not only require substantial expenditures for repairing or rebuilding, but also can cause significant indirect social losses (e.g., delayed emergency response and business disruption). Facing a severely damaged transportation network, the state or local transportation authority must decide either to restore the damaged bridges to their original intact states or to upgrade their earthquake-resistant capacities. This decision point is a rare opportunity for the community to enhance the resilience of its transportation infrastructure system through thoughtful repair or rebuild decisions.

The idea of upgrading infrastructure facilities in post-hazard recovery is encapsulated in Building Back Better (BBB) principles originally proposed during the multinational recovery effort following the Indian Ocean Tsunami (Clinton, 2006). Mannakkara and Wilkinson (2013) established BBB principles for improving structural design to achieve efficiency and effectiveness in the post-disaster rebuilding process, and then tested these principles in the Indian Ocean Tsunami reconstruction in Sri Lanka, and the Victorian Bushfires in Australia.

At present, most studies on community resilience focus on resilience assessment, short-term post-hazard recovery modelling, and restoration prioritisation, with a view towards restoring the functionality of the community to a target condition (e.g., pre-hazard condition) quickly through efficient resource allocation in the recovery process. For instance, Cimellaro, Reinhorn, and Bruneau (2010) provided a framework for the quantitative definition of resilience using an analytical function that may fit both technical and organisational issues; González, Dueñas-Osorio, Sánchez-Silva, and Medaglia (2016) introduced the interdependent network design problem for optimal infrastructure system restoration subject to budget, resources, and operational constraints; Lin and Wang (2017a, 2017b) proposed a simulation-based building portfolio recovery model to predict the functionality recovery time and recovery trajectory of both individual buildings and building portfolios following a natural hazard event, and applied the model to a mid-size community; Zhang, Wang, and Nicholson (2017) presented a resilience-based framework which systematically incorporates network topology, redundancy, traffic flow, damage level, and available resources into optimisation process of network post-hazard recovery strategy; Tabandeh, Gardoni, Murphy, and Myers (2019) incorporated the information from the recovery modelling of infrastructure and variations in the socioeconomic characteristics into a time-dependent reliability analysis in order to estimate the immediate impact on individuals' well-being and model the subsequent

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recovery. Such analyses are confined to a one-event timeframe and do not investigate the option of utilising restoration opportunities to enhance existing performance levels to mitigate a community's risk to future events.

Enhancement of the performance level reduces the risk of failure in the future but requires additional cost in design, materials and construction. An optimum trade-off between the initial cost and future risk can be determined by life-cycle cost (LCC) based analysis. LCC-based analysis was initially applied to the optimum design of structures subject to hazards more than four decades ago (Liu & Neghabat, 1972, Rosenblueth, 1976a, Rosenblueth, 1976b), and was gradually improved by other scholars (Kanda & Ellingwood, 1991, Ang & De Leon, 1997, Wen & Kang, 2001, Frangopol & Maute, 2003). In recent years, some LCC-based optimisation frameworks for making optimum maintenance schedules of civil infrastructures have been proposed successively: Bocchini and Frangopol (2011) presented a probabilistic computational framework for optimising preventive maintenance applications to bridges in a highway transportation network; Dong, Frangopol, and Saydam (2014) further proposed a probabilistic framework to establish optimum pre-earthquake retrofit plans for bridge networks to achieve sustainability objectives in order to consider increasing seismic risk due to structural deterioration, emphasising on determining retrofit timing; Tapia and Padgett (2016) proposed an LCC-based framework to identify favourable engineering solutions (e.g., steel jacket, shear keys, among others) to retrofit and repair bridges susceptible to natural hazards which ensure public safety while maximising sustainability measured by lifetime environmental, economic and social performance metrics. In these studies, however, it is tacitly assumed that each bridge in a transportation network will be restored to its intact state after each earthquake in the future, which is unable to consider the impact of possible performance upgrading of the bridges in the future on the current decision-making.

In the present paper, a continuous-time Markov decision process framework based on life-cycle cost is proposed for determining the optimum earthquake-resistant levels of the rebuilt bridges. Compared to other optimisation methodologies, the proposed one owns two distinctive features: (1) network-level LCC analyses are performed in the context of BBB principles; (2) optimum decisions are made sequentially at random time points of earthquake occurrence, and the optimum earthquake-resistant level of a rebuilt bridge depends on the performance levels of other bridges in the network. The remaining of the paper is organised as follows: firstly, the theory of continuous-time Markov decision process is briefly introduced; then, a Markovian decision framework is formulated, including definition of bridge states and restoration actions, estimation of direct and indirect economic costs, and computation of state transition probabilities; finally, a hypothetical transportation network is analysed to demonstrate the application of the framework, and some conclusions are drawn.

2. Markov decision process

Markov decision process (MDP) provides a mathematical framework for modelling sequential decision making in a wide range of situations where outcomes are determined by decisions and exogenous random information. MDP theory can date back to as early as the 1950s (Bellman, 1957). It has long been applied to different branches of civil engineering. For example, Camahan, Davis, Shahin, Keane, and Wu (1987) developed an MDP procedure for making optimum maintenance decisions for a deteriorating pavement; Tao, Corotis, and Ellis (1995) proposed an optimum structural design framework that synthesises the initial structural design and its maintenance policy over a design lifetime by combining MDP and structural reliability theory. These problems had been modelled as discrete-time Markov decision processes (DTMDP) by assuming that the decisions are made at a sequence of equal-interval time points, which is justifiable for maintenance practices. However, for the problem at hand (post-earthquake restoration), the time points of decision are random. Therefore, continuous-time Markov decision process (CTMDP) is resorted to instead.

2.1. Continuous-Time Markov decision process

Stochastic optimisation problems in which decisions are made sequentially at randomly spaced time points can be formulated by the following optimality equation (Powell, 2007):

$$\min_{\pi} \mathsf{E}\{\sum_{n=0}^{\{N|t_N\le T\}} e^{-\lambda t_n} C\Big(S_n, A(S_n, t_n|\pi)\Big)\} \tag{1}$$

where $\mathsf{E}\{\cdot\}$ is the expectation operator with respect to the randomness involved in the process which is not explicitly presented in the equation; S_n is the system state at time t_n; $A(S_n, t_n|\pi)$ is the action determined by policy π and state S_n; $C(S_n, A(S_n, t_n|\pi))$ is the cost incurred when action $A(S_n, t_n|\pi)$ is taken; λ is the discount rate that reflects the time value of money; and T is the residual time horizon. For the problem at hand, S_n denotes the earthquake-resistant level of a bridge as well as its damage state after an earthquake; $A(S_n, t_n|\pi)$ denotes the target earthquake-resistant level for restoring or rebuilding the bridge; $C(S_n, A(S_n, t_n|\pi))$ comprises not only of the direct economic loss but also of the indirect economic loss due to travel-time delay, as discussed in detail in Section 4.

The solution to Equation (1) for realistic engineering decision problems is computationally difficult or intractable. Thus, the usual method of attack is to break the problem into a set of sub-problems by dynamic programming:

$$V^*(t,S) = \min_{\pi}\{C\Big(S,A(S,t|\pi)\Big) + \sum_{S'\in\mathcal{S}}\int_t^T e^{-\lambda\tau} f(\tau) P\Big(\tau,S'|S,A(S,t|\pi)\Big) V^*(\tau,S')d\tau\}, S\in\mathcal{S} \tag{2}$$

where $f(\tau)$ denotes the probability density function of random time interval; $V^*(t,S)$ denotes the value of being in

state S at time t, which is essentially the minimum expected present value of cost from the time point t on; $P(\tau,S'|S,A(S,t|\pi))$ denotes the transition probability from state S at time t to state S' at time τ given an action $A(S,t|\pi)$, and is determined by the dynamics of the system itself as well as the probabilistic distribution of the new random information that arrives between t and τ. Note that the problems are assumed to be Markovian so that the state at the next step only depends on the current state rather than on all the previous states.

It is tacitly assumed that a completely damaged bridge will be rebuilt at the same site after the earthquake. From this viewpoint, the service life of a bridge can be seen as infinity. In addition, progressive deterioration and fluctuation of the parameters (such as discount rate) over time in the long run are temporarily not considered in this paper for simplicity. Under these circumstances, Equation (2) can be further simplified as:

$$V^*(S) = \min_{\pi}\Big(C\Big(S,A(S|\pi)\Big) + \sum_{S'\in\mathcal{S}}\left[\int_0^\infty e^{-\lambda\tau}f(\tau)d\tau\right]P\Big(S'|S,A(S|\pi)\Big)V^*(S')\Big),\quad S\in\mathcal{S} \tag{3}$$

Earthquake occurrences are modelled as a Poisson process. Thus, a set of Bellman's equations of standard form with an equivalent discount factor $\nu/(\lambda+\nu)$ is finally obtained as follows:

$$\begin{aligned} V^*(S) &= \min_{\pi}\Big(C\Big(S,A(S|\pi)\Big) + \sum_{S'\in\mathcal{S}}\left[\int_0^\infty e^{-\lambda\tau}(\nu e^{-\nu\tau})d\tau\right]P\Big(S'|S,A(S|\pi)\Big)V^*(S')\Big) \\ &= \min_{\pi}\Bigg(C\Big(S,A(S|\pi)\Big) + \sum_{S'\in\mathcal{S}}\frac{\nu}{\lambda+\nu}P\Big(S'|S,A(S|\pi)\Big)V^*(S')\Bigg), S\in\mathcal{S} \end{aligned} \tag{4}$$

where ν denotes the mean occurrence rate of earthquakes.

2.2. Policy iteration

Equation (4) is solved through policy iteration, which has good convergence properties. Policy iteration randomly selects an initial policy and then performs the following two steps iteratively: (1) given a policy, evaluate the corresponding state values; (2) with the state values known, search for a better policy. The procedure is as follows:

Step 0. Initialisation:

a). Set an initial policy π_0.

b). Set $i=1$.

Step 1. Policy evaluation:

Given policy π_{i-1}, compute the state values by solving a set of linear equations:

$$V(S|\pi_{i-1}) = C\Big(S,A(S|\pi_{i-1})\Big) + \frac{\nu}{\lambda+\nu}\sum_{S'\in\mathcal{S}}P\Big(S'|S,A(S|\pi_{i-1})\Big) \times V(S'|\pi_{i-1}), \forall S\in\mathcal{S} \tag{5}$$

Step 2. Policy improvement:

Based on the state values $\{V(S|\pi_{i-1}), S\in\mathcal{S}\}$, search for a better policy π_i :

$$A(S|\pi_i) = \arg\min_{A\in\mathcal{A}}\left(C(S,A) + \frac{\nu}{\lambda+\nu}\sum_{S'\in\mathcal{S}}P(S'|S,A)V(S'|\pi_{i-1})\right),\quad \forall S\in\mathcal{S} \tag{6}$$

Step 3. If $\pi_i=\pi_{i-1}$, set $\pi^*=\pi_i$ and stop; otherwise, set $i=i+1$ and go back to Step 1.

Rewrite Equation (5) in a compact matrix form:

$$\mathsf{V}_{\pi_{i-1}} = \mathsf{C}_{\pi_{i-1}} + \frac{\nu}{\lambda+\nu}\mathsf{P}_{\pi_{i-1}}\mathsf{V}_{\pi_{i-1}} \tag{7}$$

where

$$\mathsf{V}_{\pi_{i-1}} = \begin{bmatrix} V_{\pi_{i-1}}(S_1) \\ \vdots \\ V_{\pi_{i-1}}(S_K)\end{bmatrix}, \mathsf{C}_{\pi_{i-1}} = \begin{bmatrix} C\Big(S_1,A(S_1|\pi_{i-1})\Big) \\ \vdots \\ C\Big(S_K,A(S_K|\pi_{i-1})\Big)\end{bmatrix},$$
$$\mathsf{P}_{\pi_{i-1}} = \begin{bmatrix} P\Big(S_1|S_1,A(S_1|\pi_{i-1})\Big) & \cdots & P\Big(S_K|S_1,A(S_1|\pi_{i-1})\Big) \\ \vdots & & \vdots \\ P\Big(S_1|S_K,A(S_K|\pi_{i-1})\Big) & \cdots & P\Big(S_K|S_K,A(S_K|\pi_{i-1})\Big)\end{bmatrix} \tag{8}$$

Note that K denotes the total number of states. Thus, the policy evaluation in Step 1 can be easily done through matrix inversion:

$$\mathsf{V}_{\pi_{i-1}} = \left(\mathsf{I} - \frac{\nu}{\lambda+\nu}\mathsf{P}_{\pi_{i-1}}\right)^{-1}\mathsf{C}_{\pi_{i-1}} \tag{9}$$

where I is the identity matrix.

It is worth emphasising that the computational cost required for solving Equation (4) is proportional to the total number of system states, which increases exponentially with the number of bridges for the problem at hand (curse of dimensionality). In order to extend the applicability of the proposed methodology, it is necessary to resort to approximate solving algorithms of high efficiency in the follow-up studies, such as reinforcement learning (Bradtke & Duff, 1995) and approximate dynamic programming (Powell, Simao, & Bouzaiene-Ayari, 2012), among others.

3. Probabilistic seismic hazard analysis

In order to describe the uncertainty propagation within the proposed framework, the fundamental of probabilistic seismic hazard analysis (PSHA) is introduced in this section. PSHA is used to quantify the exceedance probabilities of different seismic ground motion intensities at specified engineering sites by taking various uncertainties in seismic source, earthquake occurrence, ground motion attenuation, and local site conditions into account (Cornell, 1968, Mcguire, 2007).

Table 1. Numbering of bridge states.

		Damage state				
		ds_1	ds_2	ds_3	ds_4	ds_5
Earthquake-resistant level	I	$S_{1,1}$	$S_{1,2}$	$S_{1,3}$	$S_{1,4}$	$S_{1,5}$
	II	$S_{2,1}$	$S_{2,2}$	$S_{2,3}$	$S_{2,4}$	$S_{2,5}$
	III	$S_{3,1}$	$S_{3,2}$	$S_{3,3}$	$S_{3,4}$	$S_{3,5}$
	IV	$S_{4,1}$	$S_{4,2}$	$S_{4,3}$	$S_{4,4}$	$S_{4,5}$
	V	$S_{5,1}$	$S_{5,2}$	$S_{5,3}$	$S_{5,4}$	$S_{5,5}$

There are different source models, including point source, line source, area source, and their combinations (Cornell, 1968), depending on the configuration of potential faults and their distances to the sites. For illustration, a point source model is adopted in this paper. Likewise, plenty of earthquake occurrence models have been proposed in the literature, such as Poisson model, Markov model, and semi-Markov model (Anagnos & Kiremidjian, 1988). As one of the most commonly used models in practice, Poisson model is adopted here. In addition, the truncated Gutenberg-Richter (G-R) law is adopted to describe the magnitude-frequency relationship (Gutenberg & Richter, 1944).

Once an earthquake occurs, the propagation of seismic waves from the hypocentre to an engineering site can be described by an attenuation model that relates the ground motion intensity to several seismological parameters, such as the earthquake magnitude, focal distance, and local site condition (Campbell, 2003, Boore & Atkinson, 2008, Bozorgnia, Hachem, & Campbell, 2010). For illustration, the Campbell attenuation model (Campbell, 2003) is adopted in this paper. Furthermore, the correlation between the logarithmic ground motion intensities at different sites owing to the same seismic source can be modelled by an exponential function (Wang & Takada, 2005):

$$\rho_{i,j} = \exp\left(-\|i-j\|/L_c\right) \tag{10}$$

where $\|i-j\|$ denotes the distance between site i and site j; L_c denotes the correlation length, its value depends on the physical property of the region under investigation.

Now, given an earthquake with moment magnitude $M_W = m$, the conditional joint probability density function (PDF) of the correlated logarithmic ground motion intensities at N sites can be described by a multi-variate normal distribution:

$$f_I(\boldsymbol{y}|m) = \frac{1}{\sqrt{(2\pi)^N|\boldsymbol{\Sigma}(m)|}} \times \exp\left(-\frac{1}{2}\left(\boldsymbol{y}-\boldsymbol{\mu}(m)\right)^T\boldsymbol{\Sigma}^{-1}(m)\left(\boldsymbol{y}-\boldsymbol{\mu}(m)\right)\right) \tag{11}$$

$$\boldsymbol{\mu}(m) = \begin{pmatrix} \mu_1(m) \\ \vdots \\ \mu_N(m) \end{pmatrix},$$

$$\boldsymbol{\Sigma}(m) = \begin{pmatrix} \rho_{1,1}\sigma_1(m)\sigma_1(m) & \cdots & \rho_{1,N}\sigma_1(m)\sigma_N(m) \\ \vdots & & \vdots \\ \rho_{N,1}\sigma_N(m)\sigma_1(m) & \cdots & \rho_{N,N}\sigma_N(m)\sigma_N(m) \end{pmatrix} \tag{12}$$

where $\mu_i(m)$ and $\sigma_i(m)$ are the mean value and standard deviation of the logarithmic ground motion intensity at site i, respectively, which can be estimated according to Campbell (2003).

Finally, the joint PDF of the logarithmic ground motion intensities can be obtained as follows:

$$f_I(\boldsymbol{y}) = \int_{M_0}^{M_u} f_I(\boldsymbol{y}|m) f_M(m)\,dm \tag{13}$$

where M_0 and M_u denote the lower bound and upper bound of moment magnitude, respectively; $f_M(m)$ denotes the PDF of moment magnitude, which can be derived from the G-R law. Note that the Campbell attenuation model is considered to be valid for estimating ground motion intensities for a hard-rock site. For other site conditions, the estimates need to be modified using empirical or theoretical site factors.

4. LCC-based Markovian decision framework

This section is divided into three parts: (1) definition of bridge states and restoration actions; (2) estimation of direct and indirect economic costs; (3) computation of state transition probabilities.

4.1. Definition of bridge states and restoration actions

For a group of bridges, its state space can be expressed as the Cartesian product of the state spaces of individual bridges. The same holds for its action space. In this paper, the state of a bridge is determined by its earthquake-resistant level as well as the damage state. The design response spectrum for buildings is determined by the pseudo-spectral accelerations (PSA) at short periods and 1-sec period associated with the risk-adjusted maximum considered earthquake (MCE_R) in the U.S. (ASCE, 2017). In addition, it is suggested that the collapse probability of a building due to the MCE_R be limited to 10% on average (FEMA, 2009). In view of this, the collapse probability under the MCE_R is taken as a measure of the earthquake-resistant capacity of a bridge. Without loss of generality, the MCE_R at 1-sec period is adopted. In addition, five earthquake-resistant levels (I, II, III, IV, V) have been defined, corresponding to collapse probabilities of 20%, 10%, 5%, 2.5%, and 1.25%, respectively. It is noted that the number of optional earthquake-resistant levels and the corresponding collapse probabilities are quite flexible. They can be adjusted without difficulty according to the problem at hand.

Five damage states are defined for bridges in Hazus-MH (FEMA, 2010): none (ds_1), slight (ds_2), moderate (ds_3), extensive (ds_4), and complete (ds_5). Based on the five damage states and the five earthquake-resistant levels, a total of 25 bridge states are defined, as listed in Table 1. Given a bridge state after an earthquake, the optional restoration actions are denoted by the corresponding post-decision states. Although an action space can be complicated, say by considering damage accumulation or structural retrofit, it is designed to be quite simple in the present paper for illustration. More specifically, when a bridge collapses in an earthquake, it will be rebuilt in accordance with one of the five earthquake-resistant levels; otherwise, it is restored to its

Table 2. Optional post-decision states of a bridge.

Post-earthquake state	Post-decision state
$S_{1,1}$, $S_{1,2}$, $S_{1,3}$, $S_{1,4}$	$S_{1,1}$
$S_{2,1}$, $S_{2,2}$, $S_{2,3}$, $S_{2,4}$	$S_{2,1}$
$S_{3,1}$, $S_{3,2}$, $S_{3,3}$, $S_{3,4}$	$S_{3,1}$
$S_{4,1}$, $S_{4,2}$, $S_{4,3}$, $S_{4,4}$	$S_{4,1}$
$S_{5,1}$, $S_{5,2}$, $S_{5,3}$, $S_{5,4}$	$S_{5,1}$
$S_{1,5}$, $S_{2,5}$, $S_{3,5}$, $S_{4,5}$, $S_{5,5}$	$S_{1,1}$, $S_{2,1}$, $S_{3,1}$, $S_{4,1}$, $S_{5,1}$

intact state. Hence, performance upgrading is not considered in these cases, which is usually the case in practice due to limited budget. The feasible post-decision states corresponding to different states are list in Table 2.

4.2. Estimation of direct and indirect economic costs

The costs in Equation (3) include direct economic cost for restoration as well as indirect economic cost due to travel-time delay. Given a damage state, the direct economic cost for restoring the bridge is the product of the initial construction cost and the damage ratio. The damage ratio corresponding to a damage state is the fraction of the restoration cost to the initial construction cost. The best estimates of the damage ratios used here are referred to Hazus-MH (FEMA, 2010). The initial construction cost of a bridge depends on its earthquake-resistant capacity, which is reasonable, since constructing a bridge of higher earthquake-resistant capacity requires more material and Labour. The relationship between the initial construction cost and the earthquake-resistant capacity is modelled by an empirical exponential function proposed by Ang and De Leon (1997):

$$C_{ini,k} = C_{ini,ref} \cdot \exp\left(a \cdot \left(1 - (P_{f,k}/P_{f,ref})^b\right)\right),\ k = 1,2,3,4,5 \tag{14}$$

where $P_{f,k}$, $k=1, 2, 3, 4, 5$ denote the collapse probabilities corresponding to the five earthquake-resistant levels I, II, III, IV, and V, respectively, as given in the previous section; $C_{ini,k}$, $k=1, 2, 3, 4, 5$ denote the five initial construction costs; $P_{f,ref}$ and $C_{ini,ref}$ are the collapse probability and initial construction cost of a reference design, which is simply taken as level I; a and b are set to 1.2 and 0.3, respectively, and the corresponding exponential function is depicted in Figure 1.

The indirect economic cost is assumed to be proportional to the travel-time delay experienced by travellers due to traffic disruption during the post-earthquake restoration process. The delay is usually expressed as the increase of total travel time (Furuta, Frangopol, & Nakatsu, 2011), as follows:

$$C_{ind} = \eta \sum_{i=1}^{M} \left[L_{peak} \sum_{a \in A} \left(x_a^{(i)} t_a^{(i)} - \tilde{x}_a \tilde{t}_a \right) \right] d_i \tag{15}$$

where M denotes the number of restoration stages; d_i denotes the duration of stage i; $\mathcal{A}$ denotes the set of all the links in the network; $\tilde{x}_a$ and $\tilde{t}_a$ denote the traffic flow and travel time on link a in the normal condition, while $x_a^{(i)}$ and $t_a^{(i)}$ denote the traffic flow and travel time on link a in

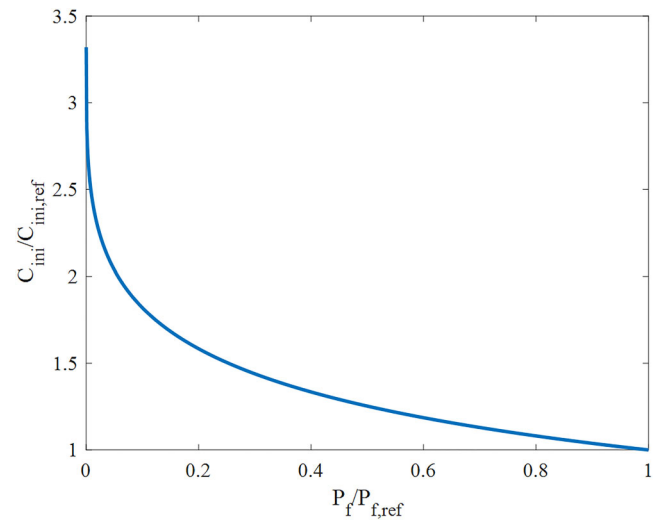

Figure 1. Exponential function model.

restoration stage i; L_{peak} denotes the duration of peak traffic in a day; η denotes the indirect economic cost coefficient, which synthesises all the negative impacts of travel-time delay on the society.

Given the traffic condition of a transportation network, the traffic flows as well as the travel times on the links can be assigned by Wardrop's user-equilibrium (UE) model (Wardrop, 1952), which assumes that travellers have complete traffic information of the transportation network, and each traveller tends to choose a route with the shortest travel time. Such individual choice will eventually reach a balance on the network level, that is, no traveller can further shorten his or her travel time by changing the route alone. The UE model can be expressed as the following optimisation problem:

$$\min_{x_a,\, a \in \mathcal{A}} \{ \sum_{a \in \mathcal{A}} \int_0^{x_a} t_a(x) dx \} \tag{16}$$

subjected to:

$$\sum_{p_{i,j} \in P_{i,j}} q_{p_{i,j}} = Q_{i,j} \tag{17}$$

$$x_a = \sum_{i \in \mathcal{V}} \sum_{j \in \mathcal{V} \cap j \neq i} \sum_{p_{i,j} \in P_{i,j}} \delta_{a,p_{i,j}} q_{p_{i,j}} \tag{18}$$

$$q_{p_{i,j}} \geq 0, \forall p_{i,j} \in P_{i,j}, \forall i \in \mathcal{V}, j \in \mathcal{V} \cap j \neq i \tag{19}$$

where $t_a(x)$ denotes the travel time on link a given a traffic flow x, which reflects the impedance of the link; $Q_{i,j}$ denotes the traffic demand between the origin-destination (O-D) pair (i, j); $\mathcal{V}$ denotes the set of all the nodes in the network; $P_{i,j}$ denotes the set of all the paths between the O-D pair (i, j); $q_{p_{i,j}}$ denotes the traffic flow assigned to route $p_{i,j}$ from $Q_{i,j}$; $\delta_{a,p_{i,j}}$ is an indicator variable: if route $p_{i,j}$ consists of link a, it is equal to 1, otherwise, it is equal to 0.

Link impedance is modelled by the widely accepted BPR function proposed by the U.S. Bureau of Public Roads here, as follows:

$$t_a(x) = t_{0,a}\left[1 + \alpha \cdot \left(\frac{x}{C_a}\right)^{\beta}\right] \tag{20}$$

where $t_{0,a}$ denotes the free flow travel time on link a; C_a denotes the traffic capacity of link a; α and β are two fitting parameters, which are taken as 0.15 and 4.0 respectively according to the suggestion of the U.S. BPR. The optimisation problem can be solved efficiently by the Frank-Wolfe algorithm (Leblanc, Morlok, & Pierskalla, 1975).

The procedure for estimating the indirect economic cost is summarised below:

Step 0. Initialisation:

a. Topological information of the network;
b. O-D traffic demands;
c. Free flow travel times $\{t_{0,a}\}_{a\in\mathcal{A}}$ and traffic capacities $\{C_a\}_{a\in\mathcal{A}}$ of all the links;
d. Damage states of the bridges;
e. Indirect economic cost coefficient η.

Step 1. Traffic assignment in the normal condition:

a. Set $i = 0$;
b. Calculate the traffic flows $\{\tilde{x}_a\}_{a\in\mathcal{A}}$ and travel times $\{\tilde{t}_a\}_{a\in\mathcal{A}}$ on all the links;

Step 2. Traffic assignment in the disrupted conditions:

a. Set $i = i + 1$;
b. According to the damage states of the bridges, discount the traffic capacities of the corresponding links;
c. Calculate the traffic flows $\{x_a^{(i)}\}_{a\in\mathcal{A}}$ and travel times $\{t_a^{(i)}\}_{a\in\mathcal{A}}$ on all the links.

Step 3. Once the restoration or rebuilding of a damaged bridge is done, update its damage state to none.
Step 4. If all the damaged bridges have been restored or rebuilt, go to the next step; otherwise, go back to Step 2.
Step 5. Estimate the indirect economic cost by Equation (15).

Note that some assumptions are made in this part: (1) damages of roads are neglected; (2) there are redundant minor roads besides the main roads, which provides detour paths once the related bridges are shut down; (3) the restoration processes of different bridges are carried out simultaneously; and (4) travel-time delay only happens during the period of peak traffic in the morning and evening of each day.

4.3. Computation of state transition probabilities

Fragility curves are modelled as lognormal distribution functions that give the probabilities of reaching or exceeding different damage states for a given ground motion intensity. For a given bridge, there are four fragility curves. Each fragility curve is characterised by a median value and an associated dispersion factor, as follows (FEMA, 2010):

$$F_k(\hat{y}) = \Phi\left(\frac{1}{\beta_k} \cdot \ln\left(\frac{\hat{y}}{\alpha_k}\right)\right), \quad k = 2, 3, 4, 5 \tag{21}$$

where $\Phi(\cdot)$ denotes the standard normal cumulative distribution function; $\hat{y}$ denotes a ground motion intensity; α_k and β_k denote the median value and dispersion factor of the fragility curve corresponding to ds_k.

According to the fragility curves, the probabilities that a bridge is in one of the five damage states after experiencing a seismic ground motion of intensity $\hat{y}$ are:

$$\begin{cases} P(\mathrm{ds}_1|\hat{y}) = 1 - \Phi\left(\frac{1}{\beta_2} \cdot \ln\left(\frac{\hat{y}}{\alpha_2}\right)\right) \\ P(\mathrm{ds}_2|\hat{y}) = \Phi\left(\frac{1}{\beta_2} \cdot \ln\left(\frac{\hat{y}}{\alpha_2}\right)\right) - \Phi\left(\frac{1}{\beta_3} \cdot \ln\left(\frac{\hat{y}}{\alpha_3}\right)\right) \\ P(\mathrm{ds}_3|\hat{y}) = \Phi\left(\frac{1}{\beta_3} \cdot \ln\left(\frac{\hat{y}}{\alpha_3}\right)\right) - \Phi\left(\frac{1}{\beta_4} \cdot \ln\left(\frac{\hat{y}}{\alpha_4}\right)\right) \\ P(\mathrm{ds}_4|\hat{y}) = \Phi\left(\frac{1}{\beta_4} \cdot \ln\left(\frac{\hat{y}}{\alpha_4}\right)\right) - \Phi\left(\frac{1}{\beta_5} \cdot \ln\left(\frac{\hat{y}}{\alpha_5}\right)\right) \\ P(\mathrm{ds}_5|\hat{y}) = \Phi\left(\frac{1}{\beta_5} \cdot \ln\left(\frac{\hat{y}}{\alpha_5}\right)\right) \end{cases} \tag{22}$$

According to the definition in Section 4.1, these mean values and dispersion factors depend on the earthquake-resistant capacity, as follows:

$$\begin{cases} \Phi\left(\frac{1}{\beta_{1,5}} \cdot \ln\left(\frac{\hat{y}^*}{\alpha_{1,5}}\right)\right) = 20\% \\ \Phi\left(\frac{1}{\beta_{2,5}} \cdot \ln\left(\frac{\hat{y}^*}{\alpha_{2,5}}\right)\right) = 10\% \\ \Phi\left(\frac{1}{\beta_{3,5}} \cdot \ln\left(\frac{\hat{y}^*}{\alpha_{3,5}}\right)\right) = 5\% \\ \Phi\left(\frac{1}{\beta_{4,5}} \cdot \ln\left(\frac{\hat{y}^*}{\alpha_{4,5}}\right)\right) = 2.5\% \\ \Phi\left(\frac{1}{\beta_{5,5}} \cdot \ln\left(\frac{\hat{y}^*}{\alpha_{5,5}}\right)\right) = 1.25\% \end{cases} \tag{23}$$

where $\hat{y}^*$ denotes the PSA at 1-sec period corresponding to the MCE_{R}. Note that there are two subscripts to the median values and dispersion factors hereinafter. The first one indicates the earthquake-resistant level of the bridge, while the second one indicates the damage state.

According to Hazus-MH (FEMA, 2010), the dispersion factor is set to 0.6 for bridges. Thus, the five median values of complete damage can be determined from Equation (23):

$$\begin{cases} \alpha_{1,5} = \dfrac{\hat{y}^*}{\exp\left(0.6 \cdot \Phi^{-1}(20\%)\right)} \\ \alpha_{2,5} = \dfrac{\hat{y}^*}{\exp\left(0.6 \cdot \Phi^{-1}(10\%)\right)} \\ \alpha_{3,5} = \dfrac{\hat{y}^*}{\exp\left(0.6 \cdot \Phi^{-1}(5\%)\right)} \\ \alpha_{4,5} = \dfrac{\hat{y}^*}{\exp\left(0.6 \cdot \Phi^{-1}(2.5\%)\right)} \\ \alpha_{5,5} = \dfrac{\hat{y}^*}{\exp\left(0.6 \cdot \Phi^{-1}(1.25\%)\right)} \end{cases} \tag{24}$$

Further, it is assumed that the ratios between the median values of different damage states are independent of the earthquake-resistant capacity of the bridge. That is:

$$\begin{cases} [\alpha_{1,2}, \alpha_{2,2}, \alpha_{3,2}, \alpha_{4,2}, \alpha_{5,2}] = \chi_{2,5} \cdot [\alpha_{1,5}, \alpha_{2,5}, \alpha_{3,5}, \alpha_{4,5}, \alpha_{5,5}] \\ [\alpha_{1,3}, \alpha_{2,3}, \alpha_{3,3}, \alpha_{4,3}, \alpha_{5,3}] = \chi_{3,5} \cdot [\alpha_{1,5}, \alpha_{2,5}, \alpha_{3,5}, \alpha_{4,5}, \alpha_{5,5}] \\ [\alpha_{1,4}, \alpha_{2,4}, \alpha_{3,4}, \alpha_{4,4}, \alpha_{5,4}] = \chi_{4,5} \cdot [\alpha_{1,5}, \alpha_{2,5}, \alpha_{3,5}, \alpha_{4,5}, \alpha_{5,5}] \end{cases} \tag{25}$$

where $(\chi_{2,5}, \chi_{3,5}, \chi_{4,5})$ are the fixed ratios. Note that in-depth studies need to be carried out in order to obtain more reliable fragility curves.

The state transition probabilities involved in Equation (4) can be computed by combining PSHA and fragility analysis. Firstly, for a single bridge, the non-zero conditional state transition probabilities are:

$$\begin{cases} P(S_{k,1}|S_{k,1},\hat{y}) = 1-\Phi\left(\dfrac{1}{\beta_{k,2}} \cdot \ln\left(\dfrac{\hat{y}}{\alpha_{k,2}}\right)\right) \\ P(S_{k,2}|S_{k,1},\hat{y}) = \Phi\left(\dfrac{1}{\beta_{k,2}} \cdot \ln\left(\dfrac{\hat{y}}{\alpha_{k,2}}\right)\right) - \Phi\left(\dfrac{1}{\beta_{k,3}} \cdot \ln\left(\dfrac{\hat{y}}{\alpha_{k,3}}\right)\right) \\ P(S_{k,3}|S_{k,1},\hat{y}) = \Phi\left(\dfrac{1}{\beta_{k,3}} \cdot \ln\left(\dfrac{\hat{y}}{\alpha_{k,3}}\right)\right) - \Phi\left(\dfrac{1}{\beta_{k,4}} \cdot \ln\left(\dfrac{\hat{y}}{\alpha_{k,4}}\right)\right), \quad k = 1,2,3,4,5 \\ P(S_{k,4}|S_{k,1},\hat{y}) = \Phi\left(\dfrac{1}{\beta_{k,4}} \cdot \ln\left(\dfrac{\hat{y}}{\alpha_{k,4}}\right)\right) - \Phi\left(\dfrac{1}{\beta_{k,5}} \cdot \ln\left(\dfrac{\hat{y}}{\alpha_{k,5}}\right)\right) \\ P(S_{k,5}|S_{k,1},\hat{y}) = \Phi\left(\dfrac{1}{\beta_{k,5}} \cdot \ln\left(\dfrac{\hat{y}}{\alpha_{k,5}}\right)\right) \end{cases} \tag{26}$$

For a group of bridges, the conditional state transition probabilities are products of the individual components, as follows:

$$P\left(\left(S_{q(1),r(1)},\ldots,S_{q(N),r(N)}\right)|\left(S_{q(1),1},\ldots,S_{q(N),1}\right),(\hat{y}_1,\ldots,\hat{y}_N)\right) = \prod_{j=1}^{N} P\left(S_{q(j),r(j)}|S_{q(j),1},\hat{y}_j\right) \tag{27}$$

where N denotes the number of bridges; $q(j)$ and $r(j)$ denote the earthquake-resistant level and damage state of bridge j, respectively. Note that the correlation between the seismic damages is ignored in this paper for simplicity. However, it is not difficult to consider such correlation as long as enough data on the bridges (e.g., maintenance and retrofit schedules, construction methods, and traffic loads, etc.) is available (Ghosh, Rokneddin, Padgett, & Dueñas-Osorio, 2014, Rokneddin, Ghosh, Dueñas-Osorio, & Padgett, 2014).

Finally, by convolving the conditional state transition probabilities with the joint PDF of the logarithmic ground motion intensities in Equation (13), the state transition probabilities can be obtained, as follows:

$$\begin{aligned} &P\left(\left(S_{q(1),r(1)},\ldots,S_{q(N),r(N)}\right)|\left(S_{q(1),1},\ldots,S_{q(N),1}\right)\right) \\ &= \int_{y_1} \cdots \int_{y_N} P\left(\left(S_{q(1),r(1)},\ldots,S_{q(N),r(N)}\right) \right. \\ &\left. |\left(S_{q(1),1},\ldots,S_{q(N),1}\right),(\hat{y}_1,\ldots,\hat{y}_N)\right) f_I(y_1,\ldots,y_N)dy_1 \cdots dy_N \\ &\forall q(j), r(j) \in \{1,2,3,4,5\}, j = 1,\ldots,N \end{aligned} \tag{28}$$

where $(\hat{y}_1,\ldots,\hat{y}_N)$ are obtained by taking the exponents of $(y_1,\ldots,y_N)$, followed by site adjustment according to the soil conditions (ASCE, 2017).

Monte Carlo simulation is employed to solve this complex integral. The procedure is as follows:

Step 0. Initialisation:

a. Three parameters in the truncated G-R model: M_0, M_u, and b;
b. Attenuation coefficients $c_1 \sim c_{13}$ in the Campbell model;
c. Locations of the potential seismic source and the bridges, and the empirical correlation length;
d. Site conditions;
e. Set $P((S_{q(1),r(1)},\ldots,S_{q(N),r(N)})|(S_{q(1),1},\ldots,S_{q(N),1})) = 0$, for $\forall q(j), r(j) \in \{1,2,3,4,5\}$, $j = 1,\ldots,N$.

Step 1. Sampling of magnitude:

Generate N_1 samples of magnitude $\{m_i\}_{i=1}^{N_1}$ from the distribution $f_M(m)$.

Step 2. Sampling of ground motion intensities:

a. For each magnitude sample m_i, obtain the corresponding conditional joint PDF $f(\boldsymbol{y}|m_i)$, and generate N_2 samples;
b. Take the exponents of the logarithmic ground motion intensities, and adjust their values according to the site conditions, giving a total of N_1N_2 samples of adjusted ground motion intensities $\{(\hat{y}_1,\ldots,\hat{y}_N)_k\}_{k=1}^{N_1N_2}$

Step 3. Update of state transition probabilities:

For each sample of adjusted ground motion intensities $(\hat{y}_1,\ldots,\hat{y}_N)_k, k = 1,\ldots,N_1N_2$:

a. Compute the conditional state transition probabilities of each bridge;
b. Compute the conditional state transition probabilities of the bridge group;

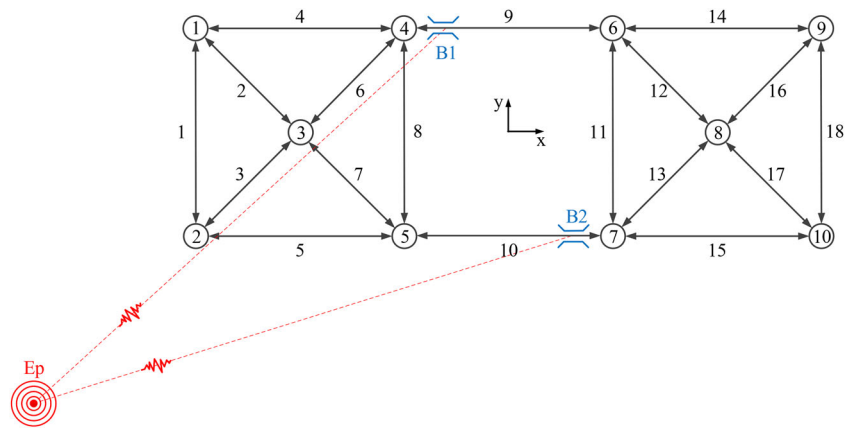

Figure 2. Hypothetical transportation network.

Table 3. Information of roads.

Road ID	L_1	L_2	L_3	L_4	L_5	L_6	L_7	L_8	L_9
Length (km)	8	5	5	6	6	5	5	8	8
Capacity (pcu/h)	1000	2000	2000	2000	2000	3000	3000	1000	3000
Peak flow (pcu/h)	300	1275	1275	825	825	2325	2325	450	2550
Road ID	L_{10}	L_{11}	L_{12}	L_{13}	L_{14}	L_{15}	L_{16}	L_{17}	L_{18}
Length (km)	8	8	5	5	6	6	5	5	8
Capacity (pcu/h)	3000	1000	3000	3000	2000	2000	2000	2000	1000
Peak flow (pcu/h)	2550	450	2325	2325	825	825	1275	1275	300

Note: pcu is abbreviation for passenger car unit.

Table 4. Peak O-D traffic demands (pcu/h).

		Destination node									
		N_1	N_2	N_3	N_4	N_5	N_6	N_7	N_8	N_9	N_{10}
Origin node	N_1	–	300	750	300	300	150	150	150	150	150
	N_2	300	–	750	300	300	150	150	150	150	150
	N_3	750	750	–	750	750	150	150	1500	150	150
	N_4	300	300	750	–	300	150	150	150	150	150
	N_5	300	300	750	300	–	150	150	150	150	150
	N_6	150	150	150	150	150	–	300	750	300	300
	N_7	150	150	150	150	150	300	–	750	300	300
	N_8	150	150	1500	150	150	750	750	–	750	750
	N_9	150	150	150	150	150	300	300	750	–	300
	N_{10}	150	150	150	150	150	300	300	750	300	–

c. Update the state transition probabilities as follows:

$$\begin{aligned} &P'\left(\left(S_{q(1),r(1)},\ldots,S_{q(N),r(N)}\right)\middle|\left(S_{q(1),1},\ldots,S_{q(N),1}\right)\right) \\ &\quad = P\left(\left(S_{q(1),r(1)},\ldots,S_{q(N),r(N)}\right)\middle|\left(S_{q(1),1},\ldots,S_{q(N),1}\right)\right) \\ &\qquad + \frac{1}{N_1N_2}P\left(\left(S_{q(1),r(1)},\ldots,S_{q(N),r(N)}\right)\right. \\ &\qquad \left.\middle|\left(S_{q(1),1},\ldots,S_{q(N),1}\right),(\hat{y}_1,\ldots,\hat{y}_N)_k\right) \\ &\qquad \forall q(j),r(j)\in\{1,2,3,4,5\},j=1,\ldots,N \end{aligned} \tag{29}$$

5. Case study: a hypothetical transportation network

5.1. Profile of the network

A hypothetical transportation network is investigated, as illustrated in Figure 2, where $N_1 \sim N_{10}$ are centroids of different traffic analysis zones (TAZ), $L_1 \sim L_{18}$ are two-way roads, B1 and B2 are bridges of the same configuration, and *Ep* is the epicentre of the potential earthquake source. The lengths, traffic capacities, and peak traffic flows of the roads are listed in Table 3. The peak O-D traffic demands between the TAZs are given in Table 4.

The epicentre is assumed to be 30 km away from the two bridges, with a focal depth of 20 km. The lower bound and upper bound of moment magnitude are taken as M_0=5.0 and M_u=8.5, with a mean occurrence period of 20 years. The *b*-value in the truncated G-R model is set to 0.8 (Petersen et al., 2008). The MCE_R at 1-sec period is assumed to be 0.6 g. The attenuation coefficients $c_1 \sim c_{13}$ is referred to Campbell (2003). The site condition of both bridges is assumed to be Class C. The ratios in Equation (25) are set to $\chi_{2,5} = 0.35/0.80$, $\chi_{3,5} = 0.45/0.80$, and $\chi_{4,5} = 0.55/0.80$, respectively. The corresponding fragility curves are depicted in Figure 3.

The discount rate is assumed to be 4%. The initial construction cost of a bridge of earthquake-resistant level I is

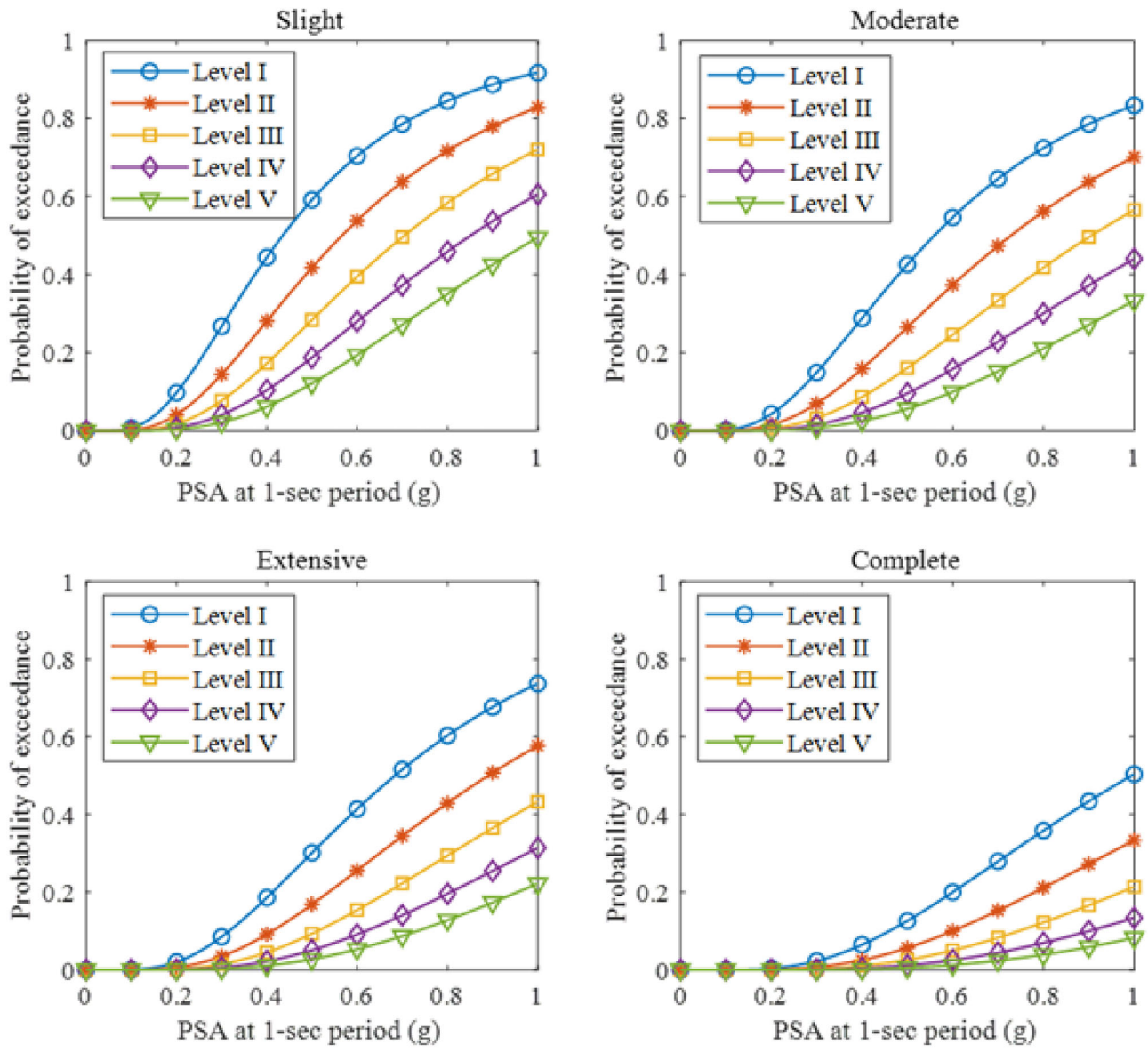

Figure 3. Fragility curves.

Table 5. Post-earthquake states in different cases.

Case 1	B1: earthquake-resistant level I, extensive damage B2: complete damage
Case 2	B1: earthquake-resistant level II, extensive damage B2: complete damage
Case 3	B1: earthquake-resistant level III, extensive damage B2: complete damage
Case 4	B1: earthquake-resistant level IV, extensive damage B2: complete damage
Case 5	B1: earthquake-resistant level V, extensive damage B2: complete damage
Case 6	B1: complete damage B2: complete damage

assumed to be \$3.0 M, and the best estimates of the damage ratios are taken as 0.03, 0.08, 0.25, and 1.00 for slight, moderate, extensive, and complete damage, respectively (FEMA, 2010). The restoration durations are assumed to be 1 day, 5 days, 90 days, and 360 days, respectively (FEMA, 2010). The daily duration of peak traffic is assumed to be 4 hrs. Note that although the costs and durations are random in practice, they are represented by deterministic values for simplicity in this study. For L_9 or L_{10}, the traffic capacity remains unchanged when B1 or B2 is slightly or moderately damaged, and a residual capacity of 30% is assumed owing to the existence of redundant routes when B1 or B2 is extensively or completely damaged.

An indirect economic cost coefficient of 82 yen/car/min (about 45 USD/pcu/hr) was adopted by Furuta et al. (2011). However, its value heavily depends on the economic development level of the region studied, and thus highly uncertain. Likewise, the value of the correlation length in Equation (10) is difficult to decide as well. In view of this, sensitivity analyses in terms of these two parameters have been carried out.

5.2. Results and discussion

Six cases described in Table 5 are considered, and the optimum earthquake-resistant level of B2 is focussed on without loss of generality. First, the optimum policy corresponding

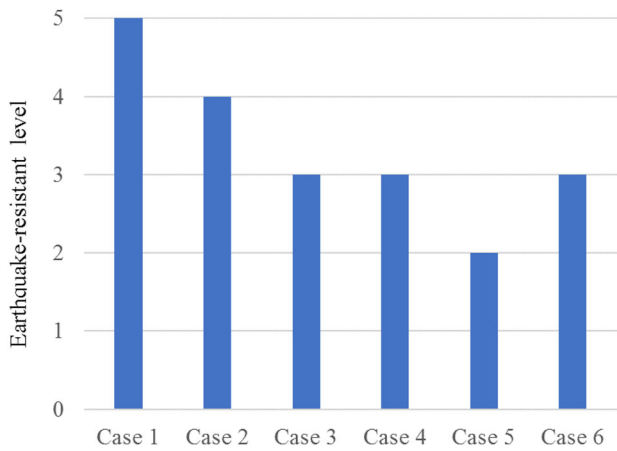

Figure 4. Optimum earthquake-resistant levels of B2.

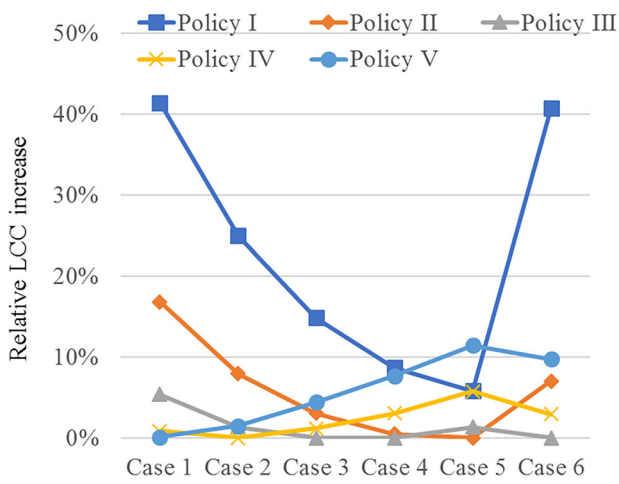

Figure 5. Relative LCC increases under different policies.

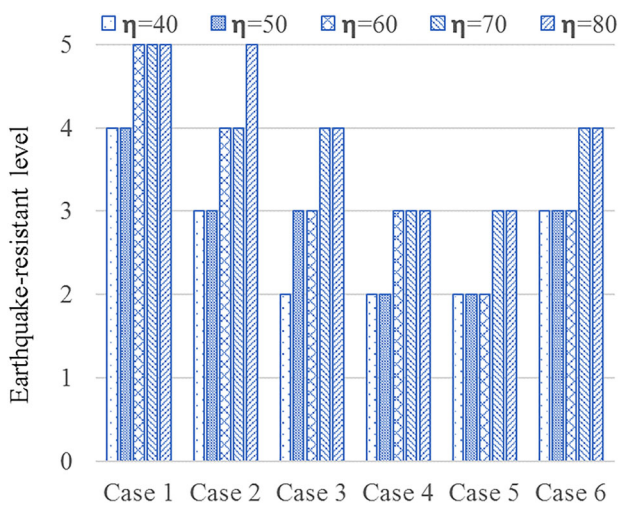

Figure 6. Optimum earthquake-resistant levels of B2 corresponding to different η.

to an indirect economic cost coefficient of 60 USD/pcu/hr and a correlation length of 30 km is obtained by the proposed approach, as depicted in Figure 4. It is observed that the optimum earthquake-resistant level for rebuilding B2 depends on the earthquake-resistance level of B1. To be specific, if B1 had been designed according to a high standard,

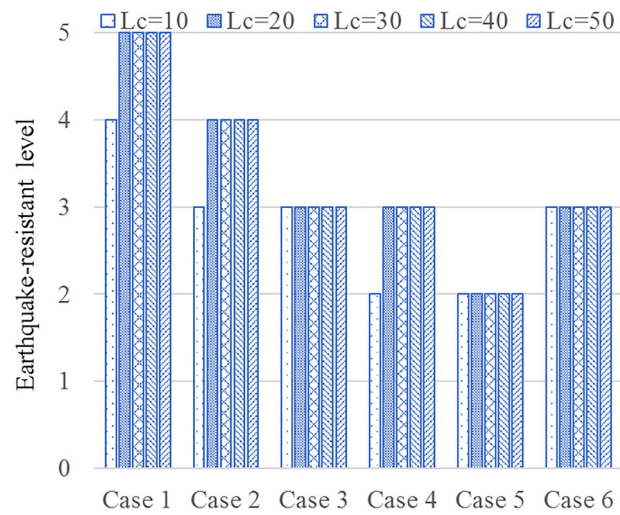

Figure 7. Optimum earthquake-resistant levels of B2 corresponding to different *Lc*.

then it is sufficient to rebuild B2 to a relatively low level, and vice versa. When both bridges are completely damaged (Case 6), it is suggested that they be rebuilt to a moderate earthquake-resistance level.

In order to demonstrate the advantage of the optimum policy, it is compared with five 'state-independent' policies in terms of the LCC of the whole transportation network, as shown in Figure 5. Policy I suggests that B2 (both bridges in Case 6) be rebuilt to earthquake-resistant level I in each case, and so on for the other four policies. The relative increases of the LCCs compared to the optimum values are depicted in Figure 5. Note that the indirect economic costs incurred at the current stage are excluded from the LCCs since they are the same for different policies. It is found that these policies lead to higher LCCs than the optimum policy. In particular, Policy I is extremely risky when the other bridge is not strong enough.

Next, for a given correlation length of 30 km, the sensitivity analysis in terms of the indirect economic cost coefficient is carried out. Depicted in Figure 6 are the optimum policies corresponding to five different indirect economic cost coefficients ranging from 40 to 80 USD/pcu/hr. It is found that the optimum decisions in Case 2 and Case 3 are more sensitive than those in the other cases. Similarly, the sensitivity analysis with respect to the correlation length for a given indirect economic cost coefficient of 60 USD/pcu/hr is also conducted. The optimum policies corresponding to five different correlation lengths ranging from 10 km to 50 km are depicted in Figure 7. It is noticed that the optimum decision in each case is insensitive to the correlation length within the above range. Therefore, in spite of its significant uncertainty, it is acceptable to regard the correlation length as a deterministic value.

Finally, the effect of network topology on the optimum policy is studied by adding a two-way road between N_5 and N_6. Three plans are considered, corresponding to the traffic capacity C^*=500, 1000, and 1500 pcu/hr, respectively, and the results are depicted in Figure 8 ($\eta = 80$ USD/pcu/hr,

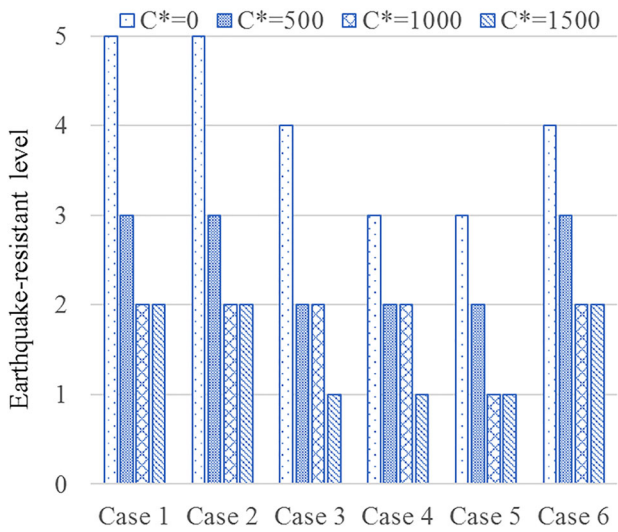

Figure 8. Optimum earthquake-resistant levels of B2 corresponding to different C^*.

L_c=30 km). Note that C^*=0 denotes the original topology. It can be seen that an extra road between the left 'island' ($N_1 \sim N_5$) and the right 'island' ($N_6 \sim N_{10}$) of the network significantly lowers the requirement for the earthquake-resistant level of B2, since it increases the redundancy of the network. As its traffic capacity increases, the optimum earthquake-resistant level gradually declines. The same holds for B1.

6. Conclusions

An LCC-based CTMDP framework for determining optimum earthquake-resistant levels of bridges in a transportation network is proposed in this paper. It is an extension to the previous LCC-based optimisation methods in two respects: (1) bridges are coupled by network-level traffic flow analysis; (2) decisions are made sequentially at time points of earthquake occurrence. It is found that the optimum earthquake-resistant level for rebuilding a completely damaged bridge depends on the capacities of other bridges in the network, which accords with our understanding of the BBB principles. That is, the functional coupling of bridges should be considered in the process of post-earthquake restoration in order to build an optimum bridge inventory in the sense of network-level LCC.

Sensitivity analyses in terms of the indirect economic cost coefficient and correlation length have been performed in the case study. A preliminary conclusion is that the value of the indirect economic cost coefficient affects the optimum policy remarkably, while the value of the correlation length has little influence on the final result. In addition, the impact of the network topology on the optimum policy is also investigated. As expected, the traffic redundancy of a transportation network plays an important role in deciding the requirement for the earthquake-resistant levels of the bridges it contains.

A major drawback of MDP lies in the exponential expansion of the computational cost with the number of bridges involved (curse of dimensionality), which hinders the application of the proposed methodology in real problems. In view of this, it is required that approximate solving algorithms of high efficiency be adopted. Although plenty of advanced algorithms have been proposed in the literature, such as reinforcement learning and approximate dynamic programming, their performance (efficiency vs. accuracy) within the proposed framework needs to be further testified. Finally, it is also necessary to further integrate the uncertainties of some critical parameters (e.g., indirect economic cost coefficient and restoration durations), as well as the mechanism of progressive deterioration into the framework.

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