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A multi-objective optimization model for retrofit strategies to mitigate direct economic loss and population dislocation

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ABSTRACT
One strategy to mitigate social and economic vulnerabilities of communities to natural disasters is to enhance the current infrastructure underlying the community. Decisions regarding allocation of limited resources to improve infrastructure components are complex and involve various trade-offs. In this study, an efficient multi-objective optimization model is proposed to support decisions regarding building retrofits within a community. In particular, given a limited budget and a heterogeneous commercial and residential building stock, solutions to the proposed model allow a detailed analysis of the trade-offs between direct economic loss and the competing objective of minimizing immediate population dislocation. The developed mathematical model is informed by earthquake simulation modeling as well as population dislocation modeling from the field of social science. The model is applied to the well-developed virtual city, Centerville, designed collaboratively by a team of engineering experts, economists, and social scientists. Multiple Pareto optimal solutions are computed in the case study and a detailed analysis regarding the various decision strategies is provided.

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1. Introduction

The built environment plays a vital role in ensuring the well-being of a community. The conditions of the physical infrastructure in the direct aftermath of an extreme hazard may have significant economic and social consequences for a community. The economic loss includes direct economic loss of building damage and long-term indirect loss (e.g. shutdown of business). The social consequences refer to many elements and include population dislocation. Dislocation has been studied as function of direct damage to residential structures (among other factors) and most residential buildings in the United States are wood-frame structures vulnerable to seismic demands. For example, the 1994 Northridge earthquake caused $40 billion in loss and the economic loss of wood-frame structures alone was over $16.2 billion (Okuyama & Chang, 2013). From 2000 to 2015, there were between 79 and 242 disaster declarations each year (FEMA, 2016). Given such high cost and frequency of large scale disruptive events, developing concepts and methods to improve the resilience of communities is an important area of research.

A significant quantity of work has been invested in the study of effective definition and metrics for community resilience (Bocchini & Frangopol, 2011; Bruneau, 2006; Hosseini, Barker, & Ramirez-Marquez, 2016). One of the more commonly used frameworks for describing resilience within communities was proposed by Bruneau et al. (2003) and includes four dimensions: the ability for communities to withstand disasters (robustness), recover quickly after a disaster (rapidity), substitute system components for others if needed (redundancy), and identify problems and mobilize resources after an event (resourcefulness). Effective pre-disaster mitigation strategies are an important element of enhancing community resilience. Current practices focus primarily on improving structural reliability of individual buildings through implementing enhanced building codes and regulations (Ellingwood, 2001; Ellingwood, Celik, & Kinali, 2007; Shinozuka, Feng, Lee, & Naganuma, 2000). However, resilience is usually recognized as a characteristic of the entire community rather than individual infrastructure elements (Bruneau et al., 2003; McAllister, 2013). The emphasis on individual buildings may be inadequate since the structures are part of a larger physical and socioeconomic system. Recent research has considered how community resilience goals can be de-aggregated to better link appropriate retrofit strategies with individual building performance (Lin, Wang, & Ellingwood, 2016). Community-based hazard mitigation policies ideally should be developed from this or other such system-wide perspectives.

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The building stock is comprised of diverse residential and non-residential buildings that facilitate commerce and the livelihood of the community. Additionally, the building inventory in a community evolves over long periods of time with varying levels of building codes and enforcement. Compliance to a single standard is not guaranteed and code levels within a community is often heterogeneous. A building retrofit strategy should account for heterogeneity of both the structural characteristics (type, age, value, code level, etc.) and purpose (e.g. residential, commercial, government). With respect to this community-based perspective we consider a theoretical optimal allocation of a limited budget to retrofit a heterogeneous building inventory under competing objectives given a scenario earthquake event.

Limited research has been conducted with respect to optimizing retrofit plans for resilience of a large-scale building inventory. Cimellaro, Reinhorn, and Bruneau (2010) considered four performance metrics relating both to the immediate effects of a disaster and the indirect effects that may be incurred during the recovery phases. The metrics cited are content loss (CL), causalities directly relating to the event, economic loss due to business interruption and relocation expenses, etc., and indirect causalities resulting from hospital dysfunction or inaccessibility. In their case study, they considered six hospitals across a region and four retrofit policies: 'no action', retrofit to life safety level, retrofit to immediate occupancy level, and rebuild. They examined the impacts to the loss metrics under the assumption that each retrofit action is applied to every hospital within the region. Jennings, van de Lindt, and Peek (2015) developed a multi-objective optimization model for retrofit strategy of wood frame buildings to minimize cost, economic loss, number of deaths, and recovery time in a community impacted by an earthquake. The multiple objectives were aggregated into a single objective using a weighting scheme and the retrofit policy was determined by use of a genetic algorithm. Using Los Angeles County, California as a case study they considered a subset of 5000 buildings and two building archetypes (two code levels for single-story residential homes) for retrofit analysis. Decisions in both studies are with respect to a relatively limited quantity of buildings and range of building types.

The present work contributes to the literature relating to retrofit multi-objective optimization along the *robustness* dimension of community resilience. While increasing robustness (or alternatively, decreasing vulnerability) represents only one dimension of the resilience concept, it may have other significant impacts others as well. In particular, financial losses to businesses and population dislocation have potentially long-term impacts that affect a communities ability to recover (e.g. loss of employment opportunities, loss of tax revenue, etc). This contribution differs from the scope of the aforementioned studies. First, the current work incorporates significant heterogeneity in terms of the building stock. The range of attributes include building purpose (e.g. residential, commercial, industrial), structure type, code status, location, and relevant socioeconomic characteristics. Secondly, the decision problem is modeled as multi-objective linear programming (LP) mathematical model. Since LP problems are computationally efficient (solvable in polynomial time), many points on and spanning the entire Pareto front are identified. Finally, given that many such solutions are determined, a detailed analysis of non-dominated strategies and trade-offs is conducted.

The proposed multi-objective retrofit optimization model allocates limited resources among all buildings in a community with respect to two competing objectives: direct economic loss and population dislocation. The optimal solutions for each structural type and community zone can provide vital information to decision makers (government leaders and other stakeholders). In particular, the model developed here serves the purpose of identifying and quantifying the vulnerabilities within a community with respect to the aforementioned objectives. While community leaders do not usually have direct ability to retrofit existing privately owned buildings, they can and do engage in a variety of strategies to indirectly affect seismic retrofits. These strategies include tax credits, permit fee reductions, low interest loans, new ordinances, public education and awareness programs, and grants (e.g. Burby, French, and Nelson, 1998; FEMA, 1994; Oregon Seismic Safety Policy Advisory Commission (OSSPAC), 2013. The approach taken by community leaders would likely be specific to the community. In this work we only provide information to support the decision of where such potential investment and/or incentives should be focused.

In the remainder of the paper, Section 2 discusses the mathematical formulation for the multi-objective optimization problem and the corresponding solution approach. Section 3 illustrates the proposed model on the well-developed virtual city, Centerville (Ellingwood, 2016). Concluding remarks and direction for future work are included in Section 4.

2. Problem formulation

2.1. Modeling approach

For the purpose of this analysis a community is assumed in which one or more distinct zones are present. A community zone is any defined geographic region that contains structures of interest. Such zones might be based on census tracts, topographically unique regions, or areas

Table 1. Summary description of building inventory in Centerville.

Structural type	Num of buildings	Zones	Code	Appraised value
W1	6190	Z2,Z3,Z4,Z5,Z6	Pre-code W2	$1, 39, 426
W2	4000	Z1,Z2,Z3,Z4	Low-code W1	$2, 39, 016
W3	50	Z1	Moderate-code W1	$3, 18, 816
W4	3196	Z1,Z2,Z3	Pre-code W1	$2, 39, 016
W5	102	Z4,Z6	Low-code W2	$39, 18, 960
W6	1352	Z7	Low-code MH	$61, 800
S1	45	Z8,Z9	Low-code S2L	$51, 34, 500
RC1	32	Z8, Govt	Low-code C1L	$49, 48, 000
RM1	76	Z8,Z10	Pre-code RM1L	$22, 05, 250
S2	6	Z9	Low-code S3	$77, 38, 750
S3	25	Z10	Pre-code S2L	$73, 82, 000
S4	45	Z11	Moderate-code S2L	$39, 3, 05, 000
RC2	1	HC	Low-code C1M	$17, 3, 52, 000
RM2	2	Fire1,Fire2	Low-code RM1L	$11, 03, 400
RC3	4	MS1,MS2,HS	Moderate-code C1L	$90, 22, 000
RM3	4	ES1,ES2,ES3,ES4	Moderate-code RM1L	$95, 21, 000

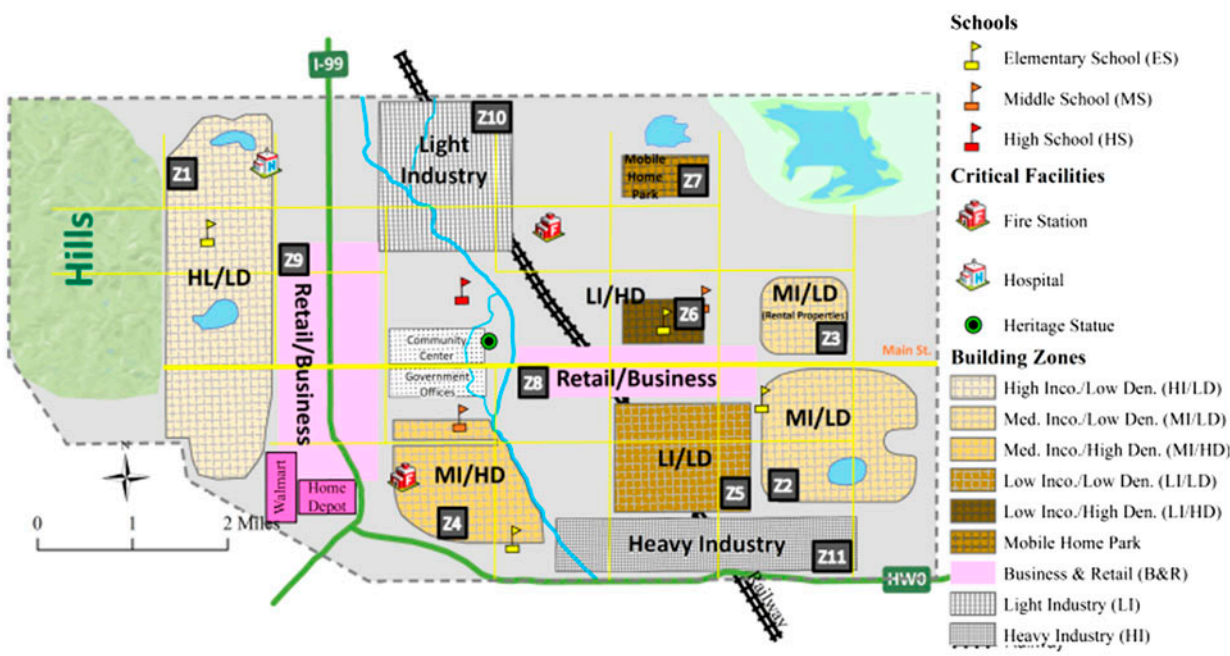

Figure 1. Centerville zoning map.

of relative homogeneity of structure types or purposes. Examples of the latter case include regions of the community that are industrial complexes, or commercial zones dominated by consumer retail outlets, or high income residential zones, etc. Furthermore, it is assumed that information associated with the structure types, building code level, occupancy type, and estimated value is available for every structure in each zone. Finally, note that the specific functional relationship between retrofit strategies and loss mitigation are specific to an earthquake hazard.

To define the optimization problem, let $\mathcal{Z}$ denote the set of community zones, $\mathcal{S}$ denote the set of structure types, and $\mathcal{K}$ denote the set of ordered code levels. Given the current building codes for each structural type in each zone, the model determines the quantities of each building type to be retrofitted to which code levels. Let the decision variable x_{ijk} denote the total number of buildings of structural type $j \in \mathcal{S}$ in zone $i \in \mathcal{Z}$ at code level $k \in \mathcal{K}$ after retrofitting. If the parameter b_{ijk} reflects the corresponding quantity of buildings prior to any mitigation efforts, the difference between x_{ijk} and b_{ijk} implies the retrofit policy. Improving the building code shifts its fragility curve so that higher magnitudes of damage are less likely for the same event. While the decision variable is logically integer, here it is modeled as a real-valued variable. Rounding of non-integer solutions is acceptable in some practical cases, in particular when the scale of the solution values is large and the relative error due to rounding is small (Miller, 2011). Since the expected building inventory is large (hundreds or thousands), the decision variable is not constrained to be integer. Solutions selected for detailed evaluation will be rounded to integer feasible retrofit actions. Finally, the decision problem in this paper is scenario-based. That is, the building inventory is subjected to a specific magnitude earthquake occurring at a specific location. Expected performance (mean value) is commonly employed as a metric to evaluate seismic loss (FEMA, 2012).

2.1.1. Objectives

Let l_{ijk} denote the expected direct economic loss due to the scenario earthquake for buildings in zone $i \in \mathcal{Z}$ of structure type $j \in \mathcal{S}$ which are at code level $k \in \mathcal{K}$. The expected loss is a function of the appraised value of the structure as well as four specific components: structural damage (SD), nonstructural drift-sensitive damage (ND), nonstructural acceleration-sensitive damage (NA), and CL. The economic loss function for a building in zone $i \in \mathcal{Z}$ of structure type $j \in \mathcal{S}$ at code level $k \in \mathcal{K}$ is adapted from work completed by the Mid-America Earthquake Center (Steelman, Song, & Hajjar, 2007),

$$l_{\text{ijk}} = M_{\text{ijk}} \left(\alpha_{\text{ijk}}^{\text{SD}} \mu_{\text{ijk}}^{\text{SD}} + \alpha_{\text{ijk}}^{\text{ND}} \mu_{\text{ijk}}^{\text{ND}} + \alpha_{\text{ijk}}^{\text{NA}} \mu_{\text{ijk}}^{\text{NA}} + \alpha_{\text{ijk}}^{\text{CL}} \mu_{\text{ijk}}^{\text{CL}} \right).$$

The parameter M_{ijk} denotes the total assessed value of the associated building. The values of $\alpha_{\text{ijk}}^{\text{SD}}$, $\alpha_{\text{ijk}}^{\text{ND}}$, and $\alpha_{\text{ijk}}^{\text{NA}}$ reflect the fraction of value relating to potential SD, ND, and NA, respectively. The value $\alpha_{\text{ijk}}^{\text{CL}}$ reflects the value ratio of the building contents to the structure's assessed value. The values for μ_{ijk}^{SD}, μ_{ijk}^{ND}, μ_{ijk}^{NA}, and μ_{ijk}^{CL} reflect the mean damage ratio associated with SD, ND, NA, and CL. The mean damage ratio is defined as the ratio of expected repair cost to the overall replacement cost. Five distinct damage states are used to model the disaster impact to structures: *none*, *slight*, *moderate*, *extensive* and *complete*. The expected repair cost can be estimated by the summation of probabilities of damage states multiplying by their corresponding repair costs. The first of the two competing objectives, to minimize direct economic loss, is provided in Equation (1).

$$\min \sum_{i \in \mathcal{Z}} \sum_{j \in \mathcal{S}} \sum_{k \in \mathcal{K}} l_{\text{ijk}} x_{\text{ijk}} \tag{1}$$

The second objective relates to the number of households in the community that are dislocated as a result of the disaster. Let $\mathcal{R} \subseteq \mathcal{Z}$ denote the set of residential zones in the community. Expected dislocation quantities for residential zone $i \in \mathcal{R}$ due to earthquake damage can be estimated based on work from Girard and Peacock (1997) and a model implemented in the Mid-America Earthquake Center Seismic Loss Assessment System (MAEviz) as proposed in MAEviz (2008). MAEviz is the product of collaboration between the Mid-America Earthquake Center and the National Center for Supercomputing Applications and supported by the National Science Foundation (Mid-America Earthquake Center, 2016). The tool is designed in support of the consequence-based risk management methodology which incorporates uncertainty and societal impacts from a system-wide perspective into decision making (Abrams, Elnashai, & Beavers, 2004). Unfortunately, there is little documentation with respect to original data used to develop the dislocation model. Estimates on population dislocation obtained from predictive models will certainly have some level of error which also may vary with respect to regional specifics and hazard types.

The model is an ordinary least-squares (OLS) regression which predicts dislocation as function of SD, non-structural damage, median household income, percentage of vacant housing, and percentage of the population that is black for each residential zone. Let d_i denote the expected household dislocation for zone $i \in \mathcal{R}$. The dislocation estimate, given a certain damage level, is computed according to Equation (2) in which the parameter t_{ij} denotes the number of households per residential structure of type j in zone i; l_{ijk}^{-c} denotes the direct economic loss of a building excluding CL; the parameters, $\beta_1, \ldots, \beta_5$ are regression coefficients; and q_i, h_i, m_i, and s_i, denote the zone characteristics: percentage of black population, percentage of vacant housing units, median household income in thousand dollars, and single-family detached housing percentage, respectively. The percentage of dislocation is adjusted to 100% if it is greater than 100 and 0% if it is negative (MAEviz, 2008).

$$d_i = \sum_{j \in \mathcal{S}} \sum_{k \in \mathcal{K}} t_{ij} b_{\text{ijk}} \left(\frac{\sum_{j \in \mathcal{S}} \sum_{k \in \mathcal{K}} l_{\text{ijk}}^{-c} x_{\text{ijk}}}{\sum_{j \in \mathcal{S}} \sum_{k \in \mathcal{K}} M_{\text{ijk}} x_{\text{ijk}}} \right) \times \left(\beta_1 + \beta_2 q_i + \beta_3 h_i + \beta_4 m_i + \beta_5 s_i \right) \quad \forall i \in \mathcal{R} \tag{2}$$

The first component in Equation (2), $\sum_{j \in \mathcal{S}} \sum_{k \in \mathcal{K}} t_{ij} b_{\text{ijk}}$, is the total number of households in zone $i \in \mathcal{R}$. The fraction in Equation (2) is the percentage of total direct economic loss (excluding CL) of the building inventory in zone i to the total appraised value of pre-disaster building inventory in the same zone. The regression coefficients, $\beta_1, \ldots, \beta_5$ provided in MAEviz (2008) are .995, −.003, −.014, .011, and −.003, respectively. The optimization model discussed in this paper uses the parameter estimates as provided in MAEViz. However, we note that these parameters could be revised to reflect regional differences or new information if available. Equation (3) reflects the second optimization objective:

Table 2. Summary of OLS parameters in Centerville.

Zone	q_i: % black	m_i: median income	s_i: % single-family detached
Z1	1	\$1, 00, 000	100
Z2	16	\$85, 000	100
Z3	10	\$60, 000	100
Z4	15	\$45, 000	52
Z5	19	\$30, 000	100
Z6	37	\$15, 000	51
Z7	20	\$10, 000	0

minimize the total population dislocation.

$$\min \sum_{i \in \mathcal{R}} d_i \tag{3}$$

The two objectives defined in Equations (1) and (3) here reflect likely conflicting objectives given limited resources. The non-residential building inventory will likely have higher appraised values than the residential buildings, however retrofit decisions on such structures will have no effect on dislocation. Given limited resources (e.g. a budget level to implement or otherwise incentivize retrofits), an allocation which emphasizes non-residential structures will likely improve the first objective at the expense of the second. This will be discussed extensively in Section 3.

Both objectives presented implicitly reflect life safety considerations. Equations (1) and (3) are functions of distinct building damage states. The damage states *none*, *slight* and *moderate* are directly tied to structural performance levels of *operational*, *immediate occupancy* and *life safety*, respectively, which are all above and beyond the minimum code requirement that ensures life safety. Furthermore, these three damage states are associated with less economic loss and dislocation, while the extensive and complete damage states are life threatening and, at the same time, result in more economic loss and population dislocation. By simultaneously attempting to minimize with respect to direct loss and population dislocation at the community level, the optimization strategy, in its essence, strategically allocates limited resources to bring as many buildings as possible up to the none, slight and moderate damage states through retrofit, which consequently protect lives in the community as a whole.

2.1.2. Constraints

Van Zandt et al. (2009) and Zhang and Peacock (2009) among others note that resilience is not homogeneous among the population within a community. In particular, households that are at a lower socioeconomic level are often more vulnerable to disasters and require a longer time to recovery from disasters (Highfield, Peacock, & Van Zandt, 2014). Such inequities are caused in part due to the 'trickle-down process' in housing in the US in which middle and upper income families buy newer homes, whereas the poor are relegated to owning increasing older and more vulnerable homes (Foley, 1980; Van Zandt), 2007. It is easy to see how a disaster may exacerbate the social vulnerability disparity in a community. The problem can become self-reinforcing. If is economically prudent (at least in the near-term) to invest resilience efforts in wealthier communities at the expense of the poor, then the vulnerability gap between rich and poor widens. When disaster strikes, the poor are hit harder and it becomes even less desirable (from a purely economic standpoint) to invest in the poorer communities and the socio-economic and resilience inequities continue to increase. Therefore, independent of economic incentives, a set inequity constraints are imposed to mitigate this cyclical phenomenon. The constraints ensure that the differences in expected dislocation levels *between* residential zones of different income levels are not increased.

To model this social equity motivated set of constraints, let $\mathcal{R}_h$, $\mathcal{R}_m$, $\mathcal{R}_l$ denote the sets of high-income, medium-income, and low-income residential zones, respectively. Note that $\mathcal{R} = \mathcal{R}_h \cup \mathcal{R}_m \cup \mathcal{R}_l$. Let $\bar{d}_i$ denote the expected dislocation of zone $i \in \mathcal{R}$ under the assumption that no residential building retrofits are implemented and let $\bar{D}$ denote a measure of the baseline disparity across the different income zones for the community,

$$\bar{D} = \left| \sum_{i \in \mathcal{R}_h} \bar{d}_i - \sum_{i \in \mathcal{R}_m} \bar{d}_i \right| + \left| \sum_{i \in \mathcal{R}_h} \bar{d}_i - \sum_{i \in \mathcal{R}_l} \bar{d}_i \right| + \left| \sum_{i \in \mathcal{R}_m} \bar{d}_i - \sum_{i \in \mathcal{R}_l} \bar{d}_i \right|$$

The constraint on dislocation inequity ensures that the retrofit actions to be undertaken do not increase this overall level of disparity, i.e.

$$\left| \sum_{i \in \mathcal{R}_h} d_i - \sum_{i \in \mathcal{R}_m} d_i \right| + \left| \sum_{i \in \mathcal{R}_h} d_i - \sum_{i \in \mathcal{R}_l} d_i \right| + \left| \sum_{i \in \mathcal{R}_m} d_i - \sum_{i \in \mathcal{R}_l} d_i \right| \leq \bar{D}$$

Six inequalities are included in the model to guarantee this social equity constraint is satisfied. When each of the six linear constraints in Equations (4)–(6) are satisfied the retrofit decisions will not increase the baseline disparity level.

$$-\frac{1}{2}\bar{D} \leq \sum_{i\in\mathcal{R}_h} d_i - \sum_{i\in\mathcal{R}_m} d_i \leq \frac{1}{2}\bar{D} \quad (4)$$

$$-\frac{1}{2}\bar{D} \leq \sum_{i\in\mathcal{R}_h} d_i - \sum_{i\in\mathcal{R}_l} d_i \leq \frac{1}{2}\bar{D} \quad (5)$$

$$-\frac{1}{2}\bar{D} \leq \sum_{i\in\mathcal{R}_m} d_i - \sum_{i\in\mathcal{R}_l} d_i \leq \frac{1}{2}\bar{D} \quad (6)$$

Given that resources to perform (or incentivize) retrofit actions are limited, let B denote a total budget available for retrofit interventions. As a simplification, the cost of a retrofit is assumed to be reflected in the assessed value of the building. That is, if a given structure type j in zone i at code level k has an assessed value of M_{ijk} and is retrofit to a higher level, $k^* > k$, then the cost of the corresponding retrofit is assumed to be the difference in the values, $M_{\mathrm{ijk}} - M_{ijk^*}$. Therefore, the budget constraint is modeled as shown in Equation (7).

$$\sum_{i\in\mathcal{Z}} \sum_{j\in\mathcal{S}} \sum_{k\in\mathcal{K}} M_{\mathrm{ijk}}(x_{\mathrm{ijk}} - b_{\mathrm{ijk}}) \leq B \quad (7)$$

In addition to the external constraints regarding social equity and budget, there are logical constraints that must be modeled. Namely, the intervention is specifically tied to improving code levels of existing buildings. Hence, the only alterable parameter in a community is the building code. Therefore, the total number of buildings in each zone of a given type must be the same before and after any retrofit policy (see Equation (8)). Likewise, every retrofit action can only improve the code level, which implies if for a given zone i and structure type j, if the buildings are originally at a code level k^*, then $x_{\mathrm{ijk}} = 0$ for all $k < k^*$ (see Equation (9)). Lastly, each decision variable can only take on only non-negative values as shown in Equation (10).

$$\sum_{k\in\mathcal{K}} x_{\mathrm{ijk}} = \sum_{k\in\mathcal{K}} b_{\mathrm{ijk}} \quad \forall i \in \mathcal{Z}, \forall j \in \mathcal{S} \quad (8)$$

$$x_{\mathrm{ijk}} = 0 \quad \forall i \in \mathcal{Z}, \forall j \in \mathcal{S}, \forall k \in \{c \in \mathcal{K} : c < k^* \text{ where } b_{ijk^*} > 0\} \quad (9)$$

$$x_{\mathrm{ijk}} \geq 0 \quad \forall i \in \mathcal{Z}, \forall j \in \mathcal{S}, \forall k \in \mathcal{K} \quad (10)$$

The mathematical model described by minimizing Equations (1) and (3) subject to the constraints described in Equations (4)–(10) is a multi-objective linear program to allocate limited resources for building retrofit to mitigate the impacts of an earthquake on a community. Since in the proposed *mitigation-based resource allocation* model for retrofit (MRA), the two objectives are conflicting, multiple Pareto optimal solutions will be identified.

2.2. Solution approach

The MRA model is a multi-objective optimization problem (MOP) and the set of optimal solutions is called the Pareto-optimal front. Each Pareto optimal solution is a non-dominated solution. A non-dominated solution is one in which no other solution exists which is superior to it with respect to all objectives. It is impossible to improve a non-dominated solution with respect to one objective without diminishing another. As such, the Pareto front can be thought of as a multi-dimensional trade-off surface. Many approaches have been developed to solve MOPs. Classical approaches convert the multiple objectives into some type of single objective. Examples include weighted sum, weighted Tchebycheff metric methods (Miettinen, 1999), Benson's method (Benson, 1978; Ehrgott, 2005), the value function method (Rosenthal, 1984), and the ϵ-constraint method (Haimes, Ladson, & Wismer, 1971; Laumanns, Thiele, & Zitzler, 2006; Mavrotas, 2009).

The weighted sum method, or scalarization technique, is one of the common and convenient classical techniques (e.g. Jennings et al., 2015). This method integrates a set of objectives into a single objective function by multiplying each objective by pre-defined weights. The weights may correspond to the decision makers preference regarding the importance of the conflicting objectives. Oftentimes, the sum of all weights is set equal to 1 and the resulting convex sum is optimized. To estimate the Pareto front, an iterative approach is used in which the weights are changed systematically and the problem is resolved many times (Koski & Silvennoinen, 1987; Marler & Arora, 2010; Zadeh, 1963). Note that designing an iterative weighting scheme is not trivial: (i) a uniform distributed set of weights does not necessarily guarantee a uniform distribution of solutions along the Pareto front and (ii) different weighting schemes can easily obtain the same optimal solutions. One drawback of aggregating multiple objectives by convex summation is that estimating the Pareto front is impossible if the feasible region in objective space is non-convex.

In the ϵ-constraint method, the MOP is reformulated by keeping one objective function f_m as the objective and modeling the others $f_1, \ldots, f_{m-1}, f_{m+1}, \ldots, f_n$ as constraints with bounds $\epsilon_1, \ldots, \epsilon_{m-1}, \epsilon_{m+1}, \ldots, \epsilon_n$. The MOP is iteratively solved for a different ϵ values. To evaluate the entire span of Pareto optimal solutions, it is necessary to determine minimum and maximum bounds for every objective function used as a constraint. This

Table 3. Summary of population dislocation in Centerville.

Zone	Income level	% Loss	Households	Dislocated	% Dislocated
Z1	High	9.29	4246	820	19.3
Z2	Medium	8.38	2267	368	16.2
Z3	Medium	7.80	800	103	12.9
Z4	Medium	12.42	4767	883	18.5
Z5	Low	12.05	1856	296	15.9
Z6	Low	10.81	4396	549	12.5
Z7	Low	12.43	1352	185	13.7

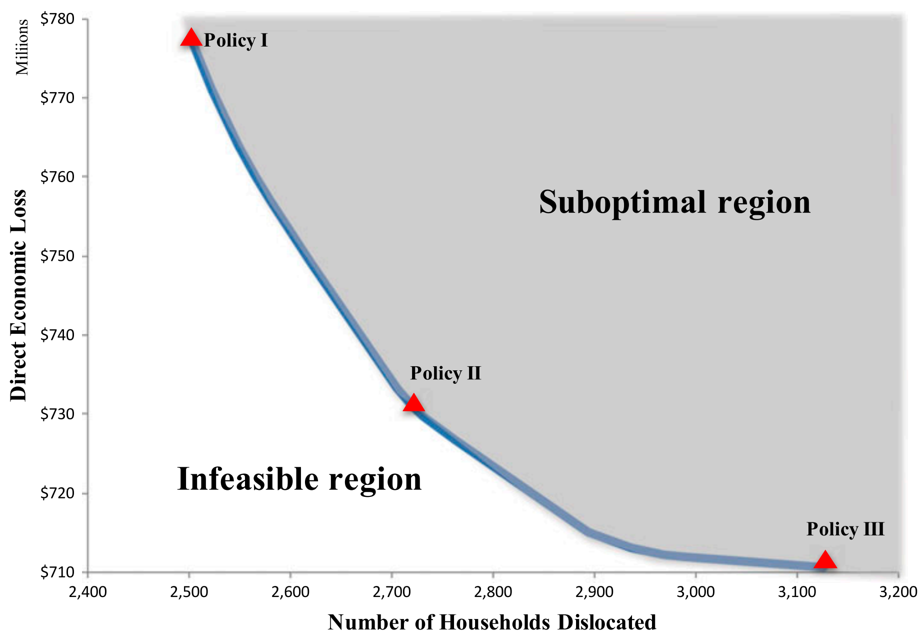

Figure 2. Pareto front: direct economic loss and dislocation ($B = \$52$M).

method can be used to estimate both convex and non-convex Pareto surfaces.

Pure MOP techniques also exist. Evolutionary algorithms are well suited to address MOPs since, in their native form, a population of solutions are managed throughout the procedure enabling a broad heuristic search for the Pareto front in a single implementation (Deb, 2001). The non-dominated sorting genetic algorithm II (Deb, Agrawal, Pratap, & Meyarivan, 2000, 2002) is among the most popular evolutionary techniques for MOPs and work in this field is expanding (e.g. Laumanns, Thiele, Deb, & Zitzler, 2002; Zhang & Li, 2007; Zitzler & Künzli, 2004). These approaches are primarily designed for complex problems which cannot be solved efficiently. Solutions determined by modern heuristics do not provide a guarantee of Pareto optimality, are not guaranteed to span the entire trade-off surface, and depending on the problem type, may require significant computation time to converge.

By careful design, the MRA model is a multi-objective linear program which can be solved exactly and iteratively using a classical approach without significant computational burden. As such, heuristic methods do not provide an advantage. The ϵ-constraint method is employed and circumvents several of the aforementioned issues, i.e. (i) the Pareto front does not have to be convex, (ii) since minimum and maximum values for both objective functions are available, the extremities of the front can be identified, and (iii) since the model is linear, exact solutions can be determined efficiently. To implement the ϵ-constraint approach, the objective function in Equation (3) is converted into the associated constraint,

$$\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{S}}\sum_{k\in\mathcal{K}} d_i \leq \epsilon \qquad (11)$$

where $\epsilon \in [\epsilon_{\min}, \epsilon_{\max}]$ and the values for $\epsilon_{\min}$ and $\epsilon_{\max}$ are determined to be the minimum possible dislocation (occurring if an ideal budget were available to retrofit every residential building) and the baseline dislocation (occurring if no retrofit actions are taken), respectively. The re-formulated problem with the single objective (1) and constraints (4) – (11) is solved many times for differ-

Table 4. Direct retrofit effects.

Policy	Dislocation % Decrease	Direct loss % Decrease
I	21.9	9.4
II	14.7	14.7
III	2.4	17.1

ent values of $\epsilon \in [\epsilon_{\min}, \epsilon_{\max}]$. The resulting solutions are Pareto optimal.

3. Case study: Centerville mitigation analysis

3.1. Baseline scenario

In order to illustrate the proposed methodology, the MRA model is applied to the well-developed virtual city, Centerville, which is designed collaboratively by a team of engineering experts, economists, and social scientists (Ellingwood, 2016). Centerville is a city with a population of approximately 20,000 households with a diverse building inventory comprised of over 15,000 structures. While the residents of the virtual city have a median income mirroring that of the US, there are residential areas within the city of high-income and those with low-income. The residential structures include single family units, apartment buildings, and mobile home units. Figure 1 depicts the layout of Centerville. There are 7 residential zones (Z1–Z7), 2 commercial zones (Z8, Z9) and 2 industrial zones (Z10, Z11), 1 hospital (HC), 2 fire stations (Fire1, Fire2) and 7 schools (ES1–ES4, MS1, MS2, HS). There are 16 structural types (W1–W6: wood; S1–S4: steel braced frame; RC1–RC3: concrete; RM1–RM3: reinforced masonry). Each of which has an assigned code level based on codes defined in HAZUS (1997). The number of buildings of each structure type, the associated code levels and zones, and appraised values are reported in Table 1. The OLS values for the independent variables associated with each residential zone are listed in Table 2. Note that the percentage of vacant units for each zone is equal to 0% and not included in the table.

The scenario event simulated is an earthquake of magnitude of 7.8 with an epicenter distance 35 kilometers southwest from the center of Centerville. The detailed calculation of the direct economic loss and SD loss can be found in Lin and Wang (2016). The total baseline direct economic loss (without retrofit intervention) is \$856M. More than half of the loss comes from damage to commercial or industrial buildings (\$434M) even though such structures represent less than 2% of the building stock. In particular, the highest zone direct economic loss of \$233M occurs in Z11. The direct economic loss of Centerville is mainly driven by non-residential buildings.

The number of households dislocated by zone under the same scenario and without retrofit is reported in Table 3. The table reports the zone, associated income level, percent of economic loss (structural and non-structural loss), number of households residing in the zone, expected dislocation quantities, and dislocation rate. The highest rates of dislocation occur in the highest income zone Z1 and in the medium income zone Z4. After discussing this with researchers involved with the development of MAEviz, it was determined that the dislocation model does not distinguish between voluntary and involuntary dislocation and such behavior would be expected. The baseline number for total expected household dislocation is 3204. The expected dislocation values for high, medium, and low-income levels are 820, 1354, and 1029, respectively. Hence, the absolute difference of dislocated households among different income levels after mitigation should be less than or equal to 1068.

3.2. Retrofit interventions

The costs associated with a retrofit implementation is assumed to be a function of the appraised value of the structure. For purposes of this study, a relatively simple relationship is employed. An improvement from code $k = 1$ (pre-code) to $k = 2$ (low-code), incurs a cost of 1% of the appraised value. Similarly, improvements from $k = 2$ to $k = 3$ (moderate code) and improvements from $k = 3$ to $k = 4$ (high-code) cost 5 and 8% of the pre-code appraised value, respectively. Three different budget levels are investigated: 15, 30, and 60% of the ideal budget, where the ideal budget is determined based on the necessary funds to retrofit every structure in Centerville to the highest code level possible.

Figure 2 depicts the Pareto front in terms of direct economic loss and dislocation for the most restrictive budget of \$52M. Over 600 solutions on the Pareto front are determined using the ϵ-constraint method. The solutions are found using Gurobi 6.0 as the LP solver and the computing time to solve each problem is around .001 s. The gray region indicates the region of dominated solutions, which are feasible but not optimal. The white region is the infeasible space, in which no solutions can satisfy the constraints (e.g. budget limitations). The triangles in Figure 2 are three solutions selected as special intervention policies for further analysis. The effect of each of the three policies are summarized in Table 4 and the solution details are reported in Table 5.

Policy I is an extreme point in the Pareto front that allocates all available resource to reduce the number of households dislocated. The corresponding number of dislocated households drops from 3204 to 2504 (a 21.8% decrease) and the direct economic loss decreases by 9.2%.

Table 5. Detail of the selected policies.

Building class	Zone	Policy I	Policy II	Policy III
Residential	Z1	1072 W2: 2 $\rightarrow$ 3 2196 W4: 1 $\rightarrow$ 2	1513 W4: 1 $\rightarrow$ 2	
	Z2	767 W1: 1 $\rightarrow$ 3		
	Z4	2567 W1: 1 $\rightarrow$ 3 25 W5: 2 $\rightarrow$ 3	2567 W1: 1 $\rightarrow$ 3 25 W5: 2 $\rightarrow$ 3	25 W5: 2 $\rightarrow$ 3
	Z6	59 W5: 2 $\rightarrow$ 3	17 W5: 2 $\rightarrow$ 3	
Commercial				
	Z8		16 S1: 2 $\rightarrow$ 3 11 RC1: 2 $\rightarrow$ 3	16 S1: 2 $\rightarrow$ 3 11 RC1: 2 $\rightarrow$ 3 30 RM1: 1 $\rightarrow$ 3
	Z9		29 S1: 2 $\rightarrow$ 3 13 RC1: 2 $\rightarrow$ 3 46 RM1: 1 $\rightarrow$ 3 6 S2: 2 $\rightarrow$ 4	29 S1: 2 $\rightarrow$ 3 13 RC1: 2 $\rightarrow$ 4 46 RM1: 1 $\rightarrow$ 3 6 S2: 2 $\rightarrow$ 4
Industrial	Z10		3 S3: 1 $\rightarrow$ 3	25 S3: 1 $\rightarrow$ 3
	Z11			7 S4: 3 $\rightarrow$ 4
Other	Fire1		1 RM2: 2 $\rightarrow$ 3	1 RM2: 2 $\rightarrow$ 3
	Fire2		1 RM2: 2 $\rightarrow$ 3	1 RM2: 2 $\rightarrow$ 3
	Govt		8 RC1: 2 $\rightarrow$ 3	8 RC1: 2 $\rightarrow$ 4

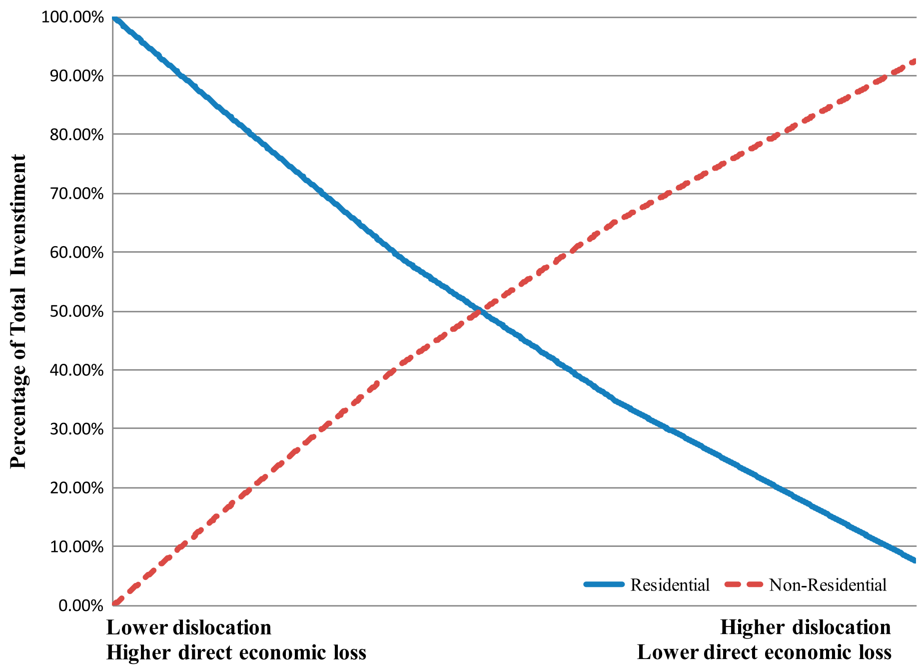

Figure 3. Investment decision: residential vs. non-residential ($B = \$52M$).

Table 5 reveals that Policy I allocates all funds to zones Z1, Z2, Z4, and Z6 – each of which has the highest expected baseline dislocation quantities (as shown in Table 3). The policy however does not allocate resources simply based on the quantity of expected dislocation by zone. The allocation mix is non-trivial. For example, only 54% of building type W2 in zone Z1 are selected for retrofit (1024 of 2000), whereas 76% of type W5 in zone Z6 are chosen (59 out of 77). The other extreme solution is Policy III, in which the number of households dislocated only drops by 2.4% but the direct loss is decreased by 17.1% (a savings of approximately \$145M over the baseline). This policy emphasizes the economic aspect by assigning most resources to non-residential zones. For instance, the 7 buildings selected in Z11 account for 26% of the available budget. Policy II is a balanced approach that allocates resources to both residential and non-residential zones.

The shift in intervention strategy present in the three selected policies is representative of the overall pattern of shifting resources from residential to non-residential structure across the Pareto front. This is visualized in Figure 3. The vertical axis denotes the percentage of budget investment and the horizontal axis reflects the spectrum of Pareto solutions ranging from the extremes of Policy I focusing on dislocation to the extremes of Policy III emphasizing economic value. The two curves are

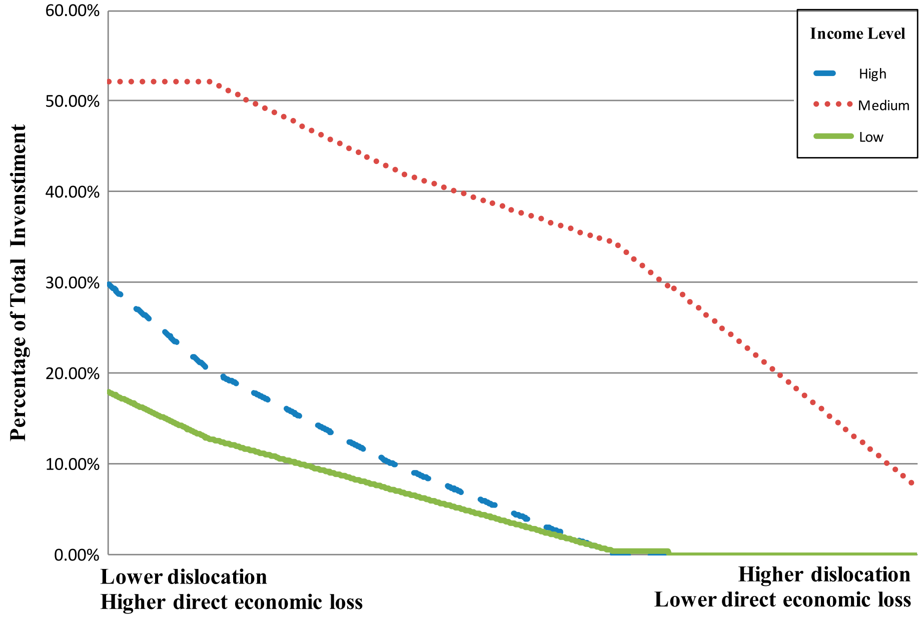

Figure 4. Investment decision: high, medium, low income residential zones ($B = \$52M$).

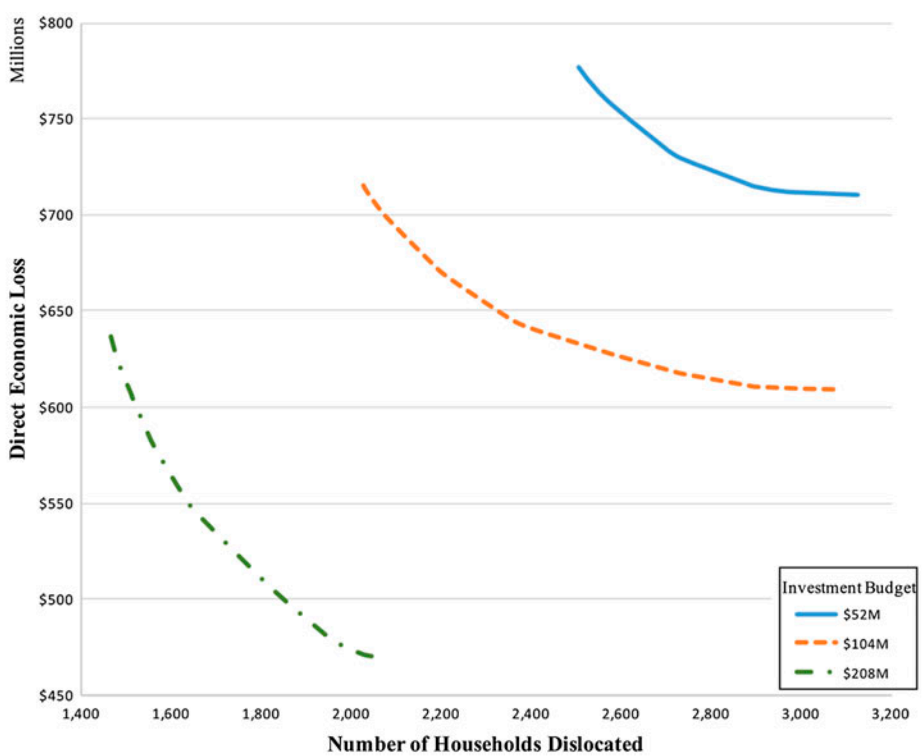

Figure 5. Pareto front: direct economic loss and dislocation ($B = \$52M, \$104M, \$208M$).

associated with budget allocation to either residential or non-residential zones.

Figure 4 provides a break out of the residential investment shown in Figure 3 by residential zone income levels. The dashed line depicts investments in the high-income zone (Z1), the dotted line reflects investments in the medium-income zones (Z2–Z4), and the solid line represents investments in the low-income zones (Z5–Z7). The MRA model consistently allocates more budget to the medium-income zones. Comparing the slope of the lines in the figure helps to establish budget priorities regarding optimal trade-offs. For instance, if a decision maker is considering shifting a proportion of resources to mitigate direct economic loss as opposed to an extreme

Table 6. Pareto optimal extreme solution objectives.

Budget		Minimum	Maximum	Range
$52M	Dislocation	2504	3126	622
	$ Loss (millions)	711	777	66
$104M	Dislocation	2026	3083	1057
	$ Loss (millions)	609	715	106
$208M	Dislocation	1467	2071	604
	$ Loss (millions)	470	636	166

policy that minimizes only dislocation (e.g. Policy I), the dashed line has the most negative slope indicating that funds should be diverted from high-income zones first, whereas funds associated with medium-income zones should be untouched (note the initial plateau on the dotted curve).

If the available retrofit budget were increased, the infeasible region in the objective space would shrink. The higher budget levels of 30 and 60% of the ideal would shift the Pareto front toward lower dislocation and economic loss values. The corresponding curves are shown in Figure 5. The solid line, dashed line, and dash-dot line are the Pareto fronts associated with the restrictive budget of $52M, the medium budget of $104M, and high level budget of $208M, respectively. With increasing budget levels more options are available. However, it is important to note that increasing budget levels do not imply a wider range of non-dominated solutions. Consider for example the far right point on the $208M Pareto front: approximately 2070 dislocated households and $470M in direct loss. All solutions at this budget level with more than 2070 dislocated households are dominated solutions. That is, no trade-off exists between the objectives that can reduce the direct loss value further. However, considering a solution with 2070 dislocations along the $104M budget-level Pareto front, there are many possible trade-offs that increase dislocation and reduce direct loss.

Comparing results across budget levels can further improve decision making. For example, the minimum population dislocation with the most restrictive budget is 2504 households. For an increase of an additional $52M, this can be reduced by 478 expected dislocations. This more costly extreme solution also comes with a reduction in direct economic loss of $61.6M.

Additionally, the various Pareto fronts shown in Figure 5 have distinctive shapes which represent interesting distinctions in the possible trade-offs. The ranges for each objective function also change base on budget level. Table 6 reports the minimum and maximum Pareto optimal dislocation and direct loss values associated with each budget. At the $52M budget level, a decision maker has $66M of potential loss to consider against the possibility of having 622 households dislocate. That is, the extreme trade-off implies a ratio of $1,06,000 in loss per potential dislocated household. At the highest budget level, this shifts to a differential of only 604 dislocations, but $166M of potential loss, giving a ratio of nearly $2,75,000 per potential household.

4. Conclusions

Community leaders, emergency planners, and local government representatives are faced with difficult decision problems and conflicting objectives when dealing with community resilience. The multiple objective optimization model developed in this study is meant as a decision support tool for such stakeholders as they consider investments to impact community vulnerability. Using the virtual community Centerville, the model is implemented and a sampling of the available trade-off analysis is presented. In particular, given the competing objectives of minimizing population dislocation and minimizing direct economic loss to the building stock, decisions regarding strategies to allocate funds for retrofit are complex in the presence of heterogeneous structures and residents. Allocation decisions regarding code level improvements to residential or non-residential structures is only one level of complexity. More granularity is desirable. The tool proposed in this study efficiently allows for a detailed analysis of the trade-offs involved regarding decisions made at the zone, structure type, and building code level of detail. The solution approach can handle large-scale communities with numerous building types. In the present case study, optimal decisions regarding the 15,000 structures associated with 16 structure types and 4 code levels require about one millisecond of computation time on a desktop computer. As such, we expect that much larger communities with more specific structure and/or occupancy types can evaluated in reasonable time.

The efficiency of the tool allows for exploration of a variety of circumstances such as varying budget levels. This exploration can help frame the decision problem. Additionally, accurate representation of various Pareto surfaces can improve the quality of decision making. Other exploration opportunities are also immediately possible.

For instance, while the social equity constraint modeled in Equations (4)–(6) is intended to limit the expected increase in social vulnerability between various socioeconomic levels, it is a simple matter to adjust these constraints to find solutions that decrease the existing inequities.

Any multi-objective optimization model, including the MRA, is subject to the quality of the input. The OLS dislocation model presently employed could be improved. While the dislocation model is used in the well-known MAEViz tool, there is little documentation with respect to the assumptions it is based on. There are other issues that conceptually can be improved as well, i.e. the present model has to rely on truncation if the predictive displacement percentages exceed 100% (or fall below 0%). In future work, mathematical approaches with potentially better characteristics will be examined. For example, the logistic regression model developed in Lin (2009) is one possibility. Furthermore, the immediate economic loss due to damage sustained in a disaster is only one component of the true economic impact to a community. If buildings and facilities are damaged, then industrial production may be impacted; as households dislocate, the local commercial sector may decline along with sale and property tax revenue, etc. Ideally, such cascading economic factors can be integrated into a new mitigation strategy optimization model.

Finally, an astute reader may have noticed that none of the retrofit policies analyzed in Table 5 involve any school or hospital in Centerville. While with respect to the model assumptions this is fine, in future research a broader collection of objective functions encompassing more potential social and economic values of a community will be implemented. Protection of emergency facilities and school children is obviously important. Modeling the appropriate and related performance objectives and their functional relationships with various intervention strategies however is not trivial. Besides buildings, the performance of utility and transportation networks play critical roles in community resilience, which also effect on economic loss and other societal metrics. Applied research is necessary to continue developing more realistic and broadly applicable decision algorithms to help inform and support a wide array of community stakeholders.

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References

Abrams, D., Elnashai, A., & Beavers, J. (2004). A new engineering paradigm: Consequence-based engineering. Available at: http://mae.cee.uiuc.edu/documents/cbepaper.pdf, accessed September 19, 2016.

Benson, H. (1978). Existence of efficient solutions for vector maximization problems. *Journal of Optimization Theory and Applications, 26*, 569–580.

Bocchini, P., & Frangopol, D. (2011). Computational methods in structural dynamics and earthquake engineering. In P. Bocchini, D. Frangopol, M. Papadrakakis, M. Fragiadakis, & V. Plevris. (Eds.), *Resilience-driven disaster management of civil infrastructure* (pp. 1–11). Athens: Institute of Structural Analysis and Antiseismic Research, School of Civil Engineering, National Technical University of Athens (NTUA).

Bruneau, M. (2006). Enhancing the resilience of communities against extreme events from an earthquake engineering perspective. *Journal of Security Education, 1*, 159–167.

Bruneau, M., Chang, S., Eguchi, R., Lee, G., O'Rourke, T., Reinhorn, A., ... von Winterfeldt, D. (2003). A framework to quantitatively assess and enhance the seismic resilience of communities. *Earthquake spectra, 19*, 733–752.

Burby, R., French, S., & Nelson, A. (1998). Plans, code enforcement, and damage reduction: Evidence from the northridge earthquake. *Earthquake Spectra, 14*, 59–74.

Cimellaro, P., Reinhorn, A., & Bruneau, M. (2010). Framework for analytical quantification of disaster resilience. *Engineering Structures, 32*, 3639–3649.

Deb, K. (2001). *Multi-objective optimization using evolutionary algorithms* (Vol. 16). New York, NY: Wiley.

Deb, K., Agrawal, S., Pratap, A., & Meyarivan, T. (2000). A fast elitist non-dominated sorting genetic algorithm for multi-objective optimization: NSGA-II. In *International Conference on Parallel problem solving from nature PPSN VI* (pp. 849–858). Berlin: Springer.

Deb, K., Pratap, A., Agarwal, S., & Meyarivan, T. (2002). A fast and elitist multiobjective genetic algorithm: NSGA-II. *IEEE Transactions on Evolutionary Computation, 6*, 182–197.

Ehrgott, M. (2005). *Multicriteria optimization* (2nd ed.). Berlin: Springer Science & Business Media.

Ellingwood, B. (2001). Earthquake risk assessment of building structures. *Reliability Engineering & System Safety, 74*, 251–262.

Ellingwood, B. (2016). The centerville virtual community: A fully integrated decision model of interacting physical and social infrastructure systems. *Sustainable and Resilient Infrastructure*. (Under review).

Ellingwood, B., Celik, O., & Kinali, K. (2007). Fragility assessment of building structural systems in mid-America. *Earthquake Engineering & Structural Dynamics, 36*, 1935–1952.

FEMA. (1994). *Seismic retrofit incentive programs: A handbook for local governments*. Washington, DC: Technical Report. FEMA-254, Federal Emergency Management Agency. Retreived from www.fema.gov/media-library/assets/documents/1834

FEMA. (2012). *Seismic performance assessment of buildings*. Washington, DC: Technical Report. Federal Emergency Management Agency.

FEMA. (2016). *Disaster declarations by year*. Washington, DC: Technical Report. Federal Emergency Management Agency. Retrieved from https://www.fema.gov/disasters/grid/year

Foley, D. (1980). The sociology of housing. *Annual Review of Sociology, 6*, 457–478.

Girard, C., & Peacock, W. (1997). Ethnicity and segregation: Post-hurricane relocation. In Gladwin, H., Peacock, W., & Morrow, B. (Eds.), *Hurricane andrew: Ethnicity, gender and the sociology of disasters* (pp. 191–205). London: Routledge.

Haimes, Y., Ladson, L., & Wismer, D. (1971). On a bicriterion formulation of problems of integrated system identification and system optimization. *IEEE Transactions on Systems, Man, and Cybernetics, 1*, 296–297.

HAZUS (1997). *Technical manual: Earthquake loss estimation methodology*. Washington, DC: National Institute of Building for the Federal Emergency Management Agency.

Highfield, W., Peacock, W., & Van Zandt, S. (2014). Mitigation planning: Why hazard exposure, structural vulnerability, and social vulnerability matter. *Journal of Planning Education and Research, 34*, 287–300.

Hosseini, S., Barker, K., & Ramirez-Marquez, J. (2016). A review of definitions and measures of system resilience. *Reliability Engineering & System Safety, 145*, 47–61.

Jennings, E., van de Lindt, J., & Peek, L. (2015). Multi-objective community-level seismic retrofit optimization for resiliency using engineering and socioeconomic variables. *12th International Conference on Applications of Statistics and Probability in Civil Engineering* (pp. 1–10), Vancouver, Canada.

Koski, J., & Silvennoinen, R. (1987). Norm methods and partial weighting in multicriterion optimization of structures. *International Journal for Numerical Methods in Engineering, 24*, 1101–1121.

Laumanns, M., Thiele, L., Deb, K., & Zitzler, E. (2002). Combining convergence and diversity in evolutionary multi objective optimization. *Evolutionary Computation, 10*, 263–282.

Laumanns, M., Thiele, L., & Zitzler, E. (2006). An efficient, adaptive parameter variation scheme for metaheuristics based on the epsilon-constraint method. *European Journal of Operational Research, 169*, 932–942.

Lin, P., & Wang, N. (2016). Building portfolio fragility functions to support scalable community resilience assessment and effective risk communication. *Sustainable and Resilient Infrastructure*. (Under review).

Lin, P., Wang, N., & Ellingwood, B. R. (2016). A risk de-aggregation framework that relates community resilience goals to building performance objectives. *Sustainable and Resilient Infrastructure, 1*, 1–13.

Lin, Y. (2009). *Development of algorithms to estimate post-disaster population dislocation: A research-based approach* (PhD thesis). College Station, TX: Texas A&M University.

MAEviz. (2008). *Displaced household algorithm 2: An OLS through the origin approach*. Technical Report. Retrieved from https://opensource.ncsa.illinois.edu/jira/browse/MAE-567

Marler, T., & Arora, J. (2010). The weighted sum method for multi-objective optimization: New insights. *Structural and Multidisciplinary Optimization, 41*, 853–862.

Mavrotas, G. (2009). Effective implementation of the ε-constraint method in multi-objective mathematical programming problems. *Applied mathematics and computation, 213*, 455–465.

McAllister, T. (2013). Developing guidelines and standards for disaster resilience of the built environment: A research needs assessment. *NIST technical note 1795*. Gaithersburg, MD: National Institute of Standards and Technology. Retrieved from http://nvlpubs.nist.gov/nistpubs/TechnicalNotes/NIST.TN.1795.pdf

Mid-America Earthquake Center (2016). MAE center: Creating a multi-hazard approach to engineering. Retrieved from http://http://mae.cee.illinois.edu

Miettinen, K. (1999). *Nonlinear multiobjective optimization*. (Vol. 12). International Series in Operations Research & Management Science. Boston, MA: Kluwer Academic Publishers.

Miller, H. (2011). *Optimization: Foundations and applications*. New York, NY: Wiley-Interscience.

Okuyama, Y., & Chang, S. (2013). *Modeling spatial and economic impacts of disasters*. Berlin: Springer Science & Business Media.

Oregon Seismic Safety Policy Advisory Commission (OSSPAC) (2013, February). *The Oregon resilience plan: Reducing risk and improving recovery for the next Cascadia earthquake and tsunami* (Report to the 77th Oregon legislative assembly from Oregon Seismic Safety Policy advisory commission). Retrieved from http://www.oregon.gov/OMD/OEM/osspac/docs/Oregon_Resilience_Plan_Final.pdf

Rosenthal, R. (1984). *Principles of multiobjective optimization*. Monterey, California: Technical Report, Naval Postgraduate School.

Shinozuka, M., Feng, M., Lee, J., & Naganuma, T. (2000). Statistical analysis of fragility curves. *Journal of Engineering Mechanics, 126*, 1224–1231.

Steelman, J., Song, J., & Hajjar, J. (2007). *Integrated data flow and risk aggregration for consequence-based risk management of seismic regional losses*. Technical Report, Mid-America Earthquake Center, University of Illinois at Urbana-Champaign, Urbana, IL.

Van Zandt, S. (2007). Racial/Ethnic differences in housing outcomes for first-time, low-income home buyers: Findings from a National homeownership education program. *Housing Policy Debate, 18*, 431–474.

Van Zandt, S., Peacock, W., Henry, D., Grover, H., Highfield, W., & Brody, S. (2012). Mapping social vulnerability to enhance housing and neighborhood resilience. *Housing Policy Debate, 22*, 29–55.

Zadeh, L. (1963). Optimality and non-scalar-valued performance criteria. *IEEE Transactions on Automatic Control, 8*, 59–60.

Zhang, Q., & Li, H. (2007). MOEA/D: A multiobjective evolutionary algorithm based on decomposition. *IEEE Transactions on Evolutionary Computation, 11*, 712–731.

Zhang, Y., & Peacock, W. (2009). Planning for housing recovery? Lessons learned from Hurricane Andrew. *Journal of the American Planning Association, 76*, 5–24.

Zitzler, E., & Künzli, S. (2004). Indicator-based selection in multiobjective search. In *International Conference on Parallel problem solving from Nature-PPSN VIII* (pp. 832–842). Berlin: Springer.