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A multi-criteria decision analysis approach for importance identification and ranking of network components

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ABSTRACT

Analyzing network vulnerability is a key element of network planning in order to be prepared for any disruptive event that might impact the performance of the network. Hence, many importance measures have been proposed to identify the important components in a network with respect to vulnerability and rank them accordingly based on individual importance measure. However, in this paper, we propose a new approach to identify the most important network components based on multiple importance measures using a multi criteria decision making (MCDM) method, namely the technique for order performance by similarity to ideal solution (TOPSIS), able to take into account the preferences of decision-makers. We consider multiple edge-specific flow-based importance measures provided as the multiple criteria of a network where the alternatives are the edges. Accordingly, TOPSIS is used to rank the edges of the network based on their importance considering multiple different importance measures. The proposed approach is illustrated through different networks with different densities along with the effects of weighs.

1. Introduction

In light of the more frequent and more severe disruptive events that have had large-scale impacts to society, analyzing the vulnerability of critical infrastructures has attracted many researchers. In the US, recent policy has emphasized that critical infrastructures "must be secure and able to withstand" all hazards [60], suggesting that vulnerability planning is an important component of preparedness. Many of these infrastructures have interdependencies and exhibit network behavior, i.e., they define a set of infrastructures connected by one or multiple relations.

The vulnerability of a network can be defined as the magnitude of the damage to the network given the occurrence of a specific disruption [30]. Approaches for analyzing network vulnerability can be broadly classified into four categories based on how disruptions scenarios are assessed [39]: scenario-specific, strategy-specific, simulation, and mathematical modeling approaches. *Scenario-specific* approaches evaluate the possible consequences of a specific disruption scenario (e.g., [32,56]; Rodriquez-Nunez and Garcia-Palomares, 2014). *Strategy-specific* approaches assess scenarios that follow a hypothesized sequence or strategy of disruption (e.g., [28,37,29]). *Simulation* approaches obtain an effective characterization of possible disruption impacts by evaluating a suitable number of disruption scenarios (e.g., [25,27,57]). *Mathematical modeling* approaches identify disruption scenarios with the highest potential impact to network operation relating to the loss or hardening of facilities (e.g., [14,38,44]).

Network vulnerability analyses often include determining the edges or the nodes in a network that have the most impact on the performance of the network when disrupted. In the field of reliability engineering, the quantification of the impact of components on the performance of a system is accomplished with importance measures. For networks, several importance measures have been suggested with respect to vulnerability to identify important components in a network, including primarily topological measures such as the L–M measure [33] and N–Q measure [40] by which the importance of a network component is measured by the relative drop in the network efficiency after the component is removed from the network. Other importance measures for network components include travel time and travel distance (e.g., [19,52]), cost of travel time (e.g., [27,57]), and accessibility (e.g., [12,55]), among others.

On the other hand, Nicholson et al. [41] combine scenario-specific and strategy-specific approaches to recommend six flow-based importance measures (IMs) for network components (i.e., edges) to prioritize proactive efforts to reduce the vulnerability of a geographic-based

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physical network with capacitated edges (or links) such as transportation networks. These edge-specific IMs can be used to assist decision makers in prioritizing edges for recovery after a disruptive event. As the IMs measure different characteristics of the network (e.g., some are flow-driven, some combine flow with topology), different rankings of edge importance are possible. In this work, we propose the use of a multi-criteria decision analysis approach to combine these multiple characteristics into a more holistic ranking of vulnerable network edges.

The remainder of the paper is organized as follows. Section 2 gives brief definitions and notations, an overview of network edge importance measures, and an introduction to the multi-criteria decision analysis approach (TOPSIS) along with an example. The proposed approach for prioritizing network edges is stated in Section 3. In Section 4, illustrative examples are presented for different networks with different relations among nodes and edges (i.e., densities) as well as the effects of weights for the proposed approach. Finally, concluding remarks are provided in Section 5.

2. Methodological background

In this section, we discuss the background required to develop our proposed approach.

2.1. Definitions and notation

In this work, we consider a network that can be categorized as a directed graph which is denoted by $G = (V, E)$ where V is a set of n vertices or nodes and $E \subset \{(i, j): i, j \in V, i \neq j\}$ is a set of directed edges or links. For edge $(i, j) \in E$, the initial node i is called the tail and the terminal node j is called the head. Let the flow and capacity on edge $(i, j) \in E$ denoted by x_{ij} and c_{ij}, respectively. Let P represent a finite *directed path* from a source node s to a target node t, which are connected through a set of nodes in V and one or more directed edges in E. The nodes between the source node s and the target node t are called internal nodes. The *maximum capacity of a path* is equal to the minimum capacity of all edges within the path (i.e., $min_{(i,j)\in P} c_{ij}$) [20].

The $s-t$ *max flow problem* utilizes a subset of all possible paths between s and t to route a maximum flow from s to t taking into consideration the capacity of edges. So, the $s-t$ max flow problem can be formulated as a linear programing (LP) as shown in Eqs. (1) – (3) [3], where w_{st} in the objective function, Eq. (1), denotes the maximum flow from node s to node t for any source and target node pair such that $s,t \in V$ where $s \neq t$, otherwise $w_{st}=0$ if $s=t$. Eq. (2) represents the flow-conservation constraints which assure that the flow into and out of any internal node must be equal and the flow out of the source node s and into the target node t must equal w_{st}. The capacity constraints in Eq. (3) ensure that there is no negative flow as well as flow through any edge does not exceed its capacity.

$$\max w_{st} \tag{1}$$

$$\text{s. t.} \quad \sum_{(i,j)\in E} x_{ij} - \sum_{(i,j)\in E} x_{ji} = \begin{cases} w_{st} & \text{if} \quad i = s \\ 0 & \text{if} \quad i \in V/\{s, t\} \\ -w_{st} & \text{if} \quad i = t \end{cases} \tag{2}$$

$$0 \leq x_{ij} \leq c_{ij}, \qquad i, j \in V \tag{3}$$

We solve the $s-t$ max flow problem using the push–relabel maximum flow algorithm [21] for a more CPU efficient solution.

2.2. Edge importance measures

In this work, we study the multiple flow-based IMs suggested by [41] relating to max flow paths within a network which measure the importance of networks edges based on a tangible variation (i.e., flow). Hence, six edge-specific, flow-based measures are studied to identify the most important edges for a network: All Pairs Max Flow Count (MFC), Min Cutset Count (MCC), Edge Flow Centrality (EFC), Flow Capacity Rate (FCR), Weighted Flow Capacity Rate (WFCR), and One-at-a-Time Damage Impact (DI). The most important edges proposed by these IMs can be reinforced, protected prior to any disruption, or expedited during the recovery stage. All six edge-specific and flow-based IMs are defined as follows.

The *all pairs max flow edge count* IM measures the utilization of a given edge in all $s-t$ pairs max flow problems. Accordingly, if an edge is contributing more than others to all $s-t$ pair max flow problems, then it could cause a significant impact on network performance when its capacity is affected by any disruptive event.

Definition 2.1. (*MFC*). The MFC of an edge (i, j), denoted as $I_{(i,j)}^{MFC}$, is defined in Eq. (4), where the value of $\mu_{st}(i, j)$ is defined as 1 if edge (i, j) is used in a given $s-t$ max flow problem, and 0 otherwise.

$$I_{(i,j)}^{MFC} = \frac{1}{n(n-1)} \sum_{s,t\in V} \mu_{st}(i, j) \tag{4}$$

The *min cutset count* IM quantifies the involvement of a given edge to the min cutset for all $s-t$ pairs where an $s-t$ *cut* on a graph is a partitioning of nodes into two disjoint sets S and T such that $s \in S$ and $t \in T$ and the $s-t$ *cutset* is the set of edges which starts in S and ends in T. Hence, the *mincut* of a graph is the $s-t$ *cut* with minimal capacity (i.e., the sum of the capacity of the s-*t* cutset). The *min cutset count* addresses whether or not an edge is a bottleneck. That is, if an edge (i, j) is involved in the *mincutset* for an $s-t$ pair, then it is considered a bottleneck for the corresponding max flow problem. Moreover, if the capacity of an edge (i, j) is reduced due to a disruptive event, the max flow value of that $s-t$ pair will be reduced too since $s-t$ max flow equals $s-t$ min cut [20].

Definition 2.2. (*MCC*). The MCC of an edge (i, j), denoted as $I_{(i,j)}^{MCC}$, is defined in Eq. (5), where the value of $\delta_{st}(i, j)$ is defined as 1 if edge (i, j) is a member in the $s-t$ cutset, and 0 otherwise.

$$I_{(i,j)}^{MCC} = \frac{1}{n(n-1)} \sum_{s,t\in V} \delta_{st}(i, j) \tag{5}$$

The *edge flow centrality* IM measures the contribution of a given edge to the max flow of all pairs. Hence, it measures importance based on the ratio of the total volume of flow through a given edge for all possible $s-t$ pair max flow problems to the flow of all pairs max flows.

Definition 2.3. (*EFC*). The EFC of an edge (i, j), denoted as $I_{(i,j)}^{EFC}$, is defined in Eq. (6), where $w_{st}(i, j)$ is the flow on (i, j) when the max flow is routed from s to t for all $s-t$ pair max flow problems.

$$I_{(i,j)}^{EFC} = \frac{\sum_{s,t\in V} w_{st}(i, j)}{\sum_{s,t\in V} w_{st}} \tag{6}$$

The *flow capacity rate* IM quantifies how close a given edge is to becoming a potential bottleneck based on the amount of flow through that edge and its capacity as well. As a result, if the flow through a given edge is close to its capacity, then network performance could be affected by any disruption occurred on that edge. On the other hand, if an edge is significantly underutilized concerning its capacity, then it is inherently robust to disruptive events that might reduce its capacity.

Definition 2.4. (*FCR*). The FCR of an edge (i, j), denoted as $I_{(i,j)}^{FCR}$, is defined in Eq. (7) with the ratio of max flow along (i, j) to capacity c_{ij}.

$$I_{(i,j)}^{FCR} = \frac{1}{n(n-1)} \sum_{s,t\in V} \frac{w_{st}(i, j)}{c_{ij}} \tag{7}$$

A *weighted flow capacity rate* IM measures the expected impact to the overall network performance by considering the flow capacity rate of a given edge along with the expected contribution of that edge to the max flow of all pairs. So for a given edge, it weighs each term in *flow capacity rate* by the *edge flow centrality*.

Definition 2.5. (*WFCR*). The WFCR of an edge (i, j), denoted as $I_{(i,j)}^{WFCR}$, is defined in Eq. (8).

$$I_{(i,j)}^{WFCR} = \frac{1}{n(n-1)\sum_{s,t\in V} w_{st}} \sum_{s,t\in V} \frac{[w_{st}(i,j)]^2}{c_{ij}} \tag{8}$$

The *one-at-a-time damage impact* IM is an empirical measure that computes the impact to network performance when a given edge is affected by a disruptive event. It provides the average percent change through all $s-t$ max flow problems when the capacity of edge (i, j) is reduced by 50%.

Definition 2.6. *(DI)*. The DI of an edge (i, j), denoted as $I_{(i,j)}^{DI}$, is defined in Eq. (9), where $w'_{st,ij}$ is the max flow from node s to node t when the capacity of edge (i, j) is reduced by 50% (i.e., capacity is $0.5c_{ij}$).

$$I_{(i,j)}^{DI} = \frac{1}{n(n-1)} \sum_{s,t\in V} \frac{w_{st} - w'_{st,ij}}{c_{ij}} \tag{9}$$

2.3. TOPSIS

Multi-criteria decision analysis (MCDA) methods help decision makers to evaluate and assess a set of alternatives to select a preferred alternative, categorize a set of alternatives, or rank a set of alternatives according to a particular preference, along several criteria [35]. The Technique for Order Preferences by Similarity to an Ideal Solution (TOPSIS) [26] is one such MCDA method to find a best alternative based on the idea of a compromise solution, or the alternative that is nearest to positive ideal solution and farthest from the negative ideal solution for each criterion. TOPSIS has been widely used in different applications [4] such as supply chain management and logistics (e.g., [63], [23]), design, engineering and manufacturing systems (e.g., [16,36]), business and marketing management (e.g., [2,43]), Health, Safety and Environment Management (e.g., [46,54]), human resources management (e.g., [7,13]), energy management (e.g., [6,8]), chemical engineering (e.g., [45,58]), and water resources management (e.g., [5,22]), among others.

Let $A = \{A_i | i = 1, \dots, n\}$ and $C = \{C_j | j = 1, \dots, m\}$ denote the set of alternatives and the set of criteria, respectively. In our case, A is the set of edges and C is the set of importance measures previously defined. Let $Y = \left\{y_{ij} | i = 1, \dots, n; j = 1, \dots, m\right\}$ represents the set of performance ratings which includes the performance rating of each alternative with each criteria and $\omega = \{\omega_j | j = 1, \dots, m\}$ is the set of weights of the alternatives ($\omega_j \geq 0$ and $\sum_{j=1}^{m} \omega_j = 1$). As several noncommensurate criteria with potentially different scales are combined with TOPSIS, a scaling equation is needed for the performance rating. One example is shown in Eq. (10).

$$r_{ij}(y) = \frac{y_{ij}}{\sqrt{\sum_{i=1}^{n} y_{ij}^2}}, \qquad i=1,\dots,n;\; j=1,\dots,m \tag{10}$$

The weights are then applied to the scaled performance rating, shown in Eq. (11).

$$v_{ij}(y) = \omega_j r_{ij}(y), \qquad i=1,\dots,n; j=1,\dots,m \tag{11}$$

The positive ideal solution (PIS), A^+, is determined as the collection of the most preferred weighted and scaled performance ratings for each criterion. Likewise, the negative ideal solution (NIS), A^-, is the least preferred rating. The PIS and NIS is found with Eqs. (12) and (13), respectively, where J^+ and J^- are defined as the set of benefit criteria and the set of cost criteria, respectively.

$$A^+ = \{v_1^+(y), v_2^+(y), \dots, v_m^+(y)\} = \left\{\left(\max_{1\le i\le n} v_{ij}(y) | j \in J^+\right), \left(\min_{1\le i\le n} v_{ij}(y) | j \in J^-\right)\right\} \tag{12}$$

$$A^- = \{v_1^-(y), v_2^-(y), \dots, v_m^-(y)\} = \left\{\left(\min_{1\le i\le n} v_{ij}(y) | j \in J^+\right), \left(\max_{1\le i\le n} v_{ij}(y) | j \in J^-\right)\right\} \tag{13}$$

The separation of each alternative from the PIS and the NIS is found from Euclidean distance using Eqs. (14) and (15). $D_i^+$and D_i^- are the distances between alternative A_i and the PIS and the distance between alternative A_i and the NIS, respectively.

$$D_i^+ = \sqrt{\sum_{j=1}^{m} [v_{ij}(y) - v_j^+(y)]^2}, \qquad i=1,\dots,n \tag{14}$$

$$D_i^- = \sqrt{\sum_{j=1}^{m} [v_{ij}(y) - v_j^-(y)]^2}, \qquad i=1,\dots,n \tag{15}$$

Finally, the similarity of alternative A_i to the PIS is calculated with Eq. (16), where the alternative with the largest S_i^+ value is the most preferred. The ranking of alternatives across the m criteria is found by ordering the S_i^+ from largest to smallest.

$$S_i^+ = \frac{D_i^-}{D_i^+ + D_i^-}, \qquad i=1,\dots,n \tag{16}$$

3. Proposed approach

In general, IMs produce useful information for decision makers. For example, IMs identify the most important network components according to a specific performance function or how close a given network component is to becoming a potential bottleneck based its utilization, as discussed earlier in Section 2.2. Such information could be useful in enabling more effective resource allocation, such as finding possible enhancement alternatives or specifying new component characteristics (e.g., capacity or reliability) [47]. However, IMs are generally defined based on different perspectives of network performance that produce different information for decision makers; hence, they could generate different rankings of network components based on their importance as shown in Table 1 and Fig. 3. Thus, decision makers face a challenge of combining such information from different IMs and produce a unique rank of network components [47], thus the motivation of this paper.

Different approaches have been proposed in the literature that dealt with the problem of having different ranks for network components by different IMs. Rocco et al. [51] aggregate multiple performance IMs that are derived from an energy model for the security of energy supply in the European Member States using a parametric ranking technique, ordered weighted averaging, and a non-parametric ranking technique, Copeland score. Rocco and Ramirez-Marquez [48] address the following challenges with respect to having several IMs to evaluate the importance of network components: (i) multiple ranking, (ii) multi-component importance, and (iii) multi-function importance. They propose a set of solutions to overcome these challenges based on Hasse diagram, Copeland score and multi-objective optimization. Du et al. [18] utilize TOPSIS to aggregate several different nodes centrality measures (i.e., not flow-based measures) to obtain a unique importance assessment of each node that help in identifying the influential nodes in a complex network. Rocco et al. [50] combine the ranks of network components obtained by three topology-based cascade models for network outages using a non-parametric technique based on partial order concepts to provide a unique importance ranking that identifies critical nodes in a network with no consideration of decision makers'

Table 1
Alternatives scores and ranks according to their importance.

Edge	MFC		MCC		EFC		FCR		WFCR		DI	
	Score	Rank	Score	Rank	Score	Rank	Score	Rank	Score	Rank	Score	Rank
(1,2)	0.1429	5	0.0238	9	0.1429	7	0.0762	10	0.0109	9	0.0142	11
(1,3)	0.1190	7	0.0238	10	0.2054	3	0.0782	9	0.0161	5	0.0160	10
(1,4)	0.0952	11	0.0238	12	0.0982	10	0.0655	12	0.0064	12	0.0096	12
(2,3)	0.2381	1	0.1429	2	0.0893	11	0.2381	1	0.0213	2	0.0432	1
(2,5)	0.0952	12	0.0952	4	0.1071	8	0.0952	7	0.0102	11	0.0249	4
(3,4)	0.1429	6	0.0714	6	0.0982	9	0.1310	4	0.0129	8	0.0214	7
(3,5)	0.1190	8	0.0952	3	0.1518	6	0.1012	5	0.0154	7	0.0232	5
(3,6)	0.1667	3	0.0476	8	0.1875	5	0.1000	6	0.0188	3	0.0214	8
(4,6)	0.1905	2	0.0714	5	0.1964	4	0.1310	3	0.0257	1	0.0308	3
(5,7)	0.1190	9	0.0238	11	0.2321	1	0.0688	11	0.0160	6	0.0189	9
(6,5)	0.1667	4	0.1667	1	0.0625	12	0.1667	2	0.0104	10	0.0313	2
(6,7)	0.1190	10	0.0714	7	0.2054	2	0.0913	8	0.0187	4	0.0218	6

preferences.

We propose TOPSIS as a new approach to prioritize edges within a network based on IMs from multiple perspectives. It provides a rank of the network edges according to their importance based on multiple flow-based IMs in Section 2.2.

The specific stepwise procedure for performing the proposed approach is illustrated in Fig. 1.

3.1. Small example

In this section, a numerical example is introduced to illustrate the steps of applying the proposed approach. Consider a network with seven nodes and twelve edges, as shown in Fig. 2, where edges are labeled by their capacities.

To apply the proposed approach, we follow the steps shown in Fig. 1. We start the first step by calculating the performance rating (score) of each alternative by each criterion considering the flow-based IMs in Section 2.2 with the following order of the IMs: MFC, MCC, EFC, FCR, WFCR, and DI. Table 1 provides the ranks of the edges

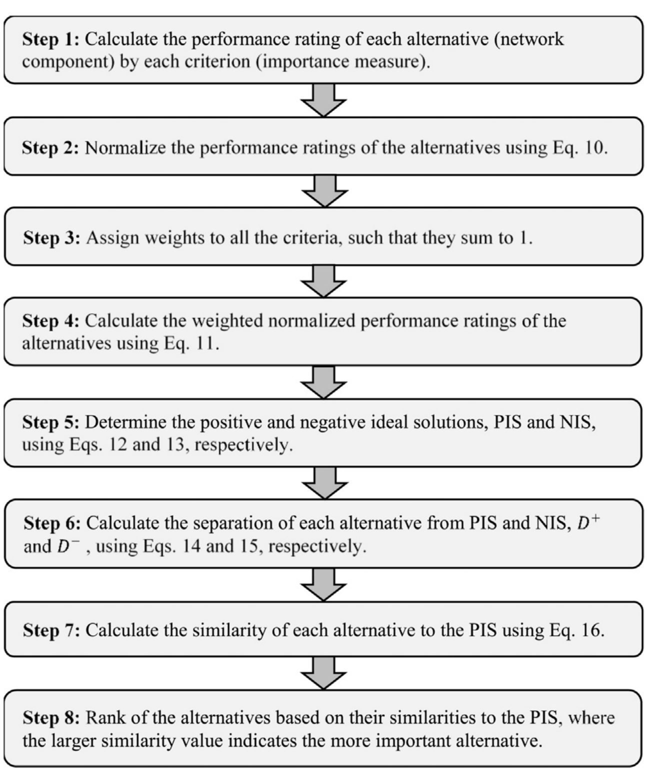

Fig. 1. The stepwise procedure of the proposed approach.

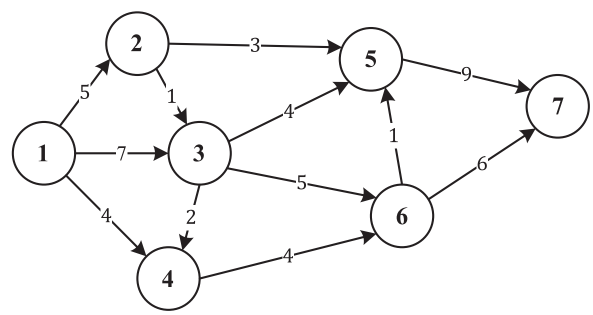

Fig. 2. A network example with labeled edges by capacity, adapted from Hillier and Lieberman [24].

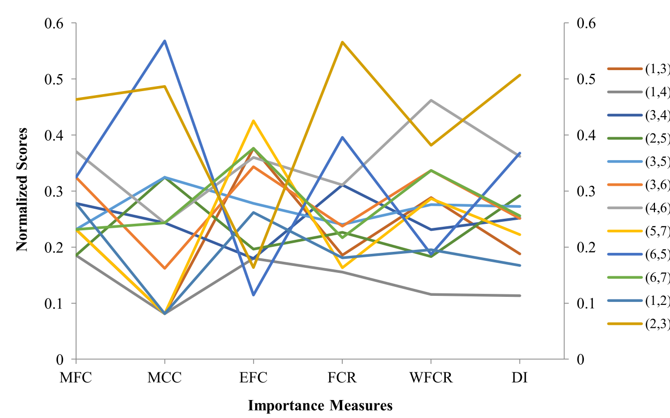

Fig. 3. Parallel coordinates plot for the edges of the network example.

obtained according to each IM perspective. Furthermore, Fig. 3 shows the Parallel Coordinates Plot (PCP) for each IM. Each line corresponds to an edge in the network example and the plotted values correspond to the normalized IM scores for the corresponding IM criteria listed on the x-axis. Each IM ranks the network edges differently as shown in Fig. 3. Note that there is no edge that can be considered as the most important for each IMs (i.e., there is no dominant edge). Also it is not clear which edge is the most important when the IM are simultaneously considered. So an aggregated importance is suggested using the proposed approach. In the second step, we normalize the performance ratings of the alternatives in Table 1 with Eq. (10). For instance, the normalized ratings of the first alternative, edge (1, 2), are $r_{1j} = \{0.2779, 0.0811, 0.2617, 0.1809, 0.1954, 0.1673\}$. After that, we assign weights to all the criteria in step 3 where for this example we consider each IM criterion to have an equal weight (i.e., the weight of each IM is 1/6). Next, we find the weighted normalized performance ratings, v_{ij}, of the alternatives in step 4 which are obtained by Eq. (11) considering equal weights for all the IMs e.g.,

Table 2
Alternatives ranks according to their similarity to the PIS.

Edge	D^+	D^-	S^+	Rank
(1,2)	0.1325	0.0334	0.2014	11
(1,3)	0.1257	0.0545	0.3025	10
(1,4)	0.1507	0.0109	0.0675	12
(2,3)	0.0476	0.1331	0.7367	1
(2,5)	0.1090	0.0546	0.3339	7
(3,4)	0.1031	0.0516	0.3335	8
(3,5)	0.0957	0.0638	0.3999	5
(3,6)	0.1026	0.0652	0.3885	6
(4,6)	0.0753	0.0953	0.5587	3
(5,7)	0.1251	0.0624	0.3326	9
(6,5)	0.0816	0.1033	0.5587	2
(6,7)	0.1002	0.0687	0.4068	4

Table 3
Alternatives ranks according to PROMETHEE.

Edge	$\varnothing^+$	$\varnothing^-$	$\varnothing$	Rank
(1,2)	0.0579	0.3291	−0.2712	11
(1,3)	0.1172	0.2640	−0.1468	10
(1,4)	0.0046	0.4902	−0.4856	12
(2,3)	0.6465	0.0843	0.5622	1
(2,5)	0.1454	0.2641	−0.1187	9
(3,4)	0.1519	0.2180	−0.0661	7
(3,5)	0.1927	0.1688	0.0238	6
(3,6)	0.2047	0.1577	0.0470	5
(4,6)	0.4164	0.0788	0.3376	2
(5,7)	0.1472	0.2577	−0.1104	8
(6,5)	0.3726	0.1922	0.1804	3
(6,7)	0.2043	0.1565	0.0477	4

v_{1j}={0.0463,0.0135,0.0436,0.0302,0.0326,0.0279}. In the fifth step, we determine the positive ideal solution, PIS, and the negative ideal solution, NIS, using Eqs. (12) and (13), respectively. So, PIS={0.0772,0.0946,0.0709,0.0942,0.0770,0.0845} and NIS={0.0309,0.0135,0.0191,0.0259,0.0192,0.0189}. The distance between each alternative and the PIS and the NIS is then calculated in step 6 by Eqs. (14) and (15), respectively, as shown in Table 2. In the following step, the similarity of each alternative to the PIS is calculated by Eq. (16). Finally, the ranks of the alternatives are determined in step 8 according to their similarity to the PIS where the largest similarity value is the most preferred alternative, see Table 2 where it shows that edge (2,3) is the most important one and edge (1,4) is the least important one.

3.2. Comparison with other MCDM methods

In this section, we compare the effectiveness of TOPSIS in the proposed approach for aggregating the different importance measures

Table 5
Alternatives ranks according to the three MCDM methods.

Edge	Rank		
	TOPSIS	PROMETHEE	CS
(1,2)	**11**	**11**	**11**
(1,3)	10	10	9
(1,4)	**12**	**12**	**12**
(2,3)	**1**	**1**	**1**
(3,2)	8	7	7
(2,5)	7	9	8
(3,5)	5	6	5
(3,6)	6	5	4
(4,6)	3	2	2
(5,7)	9	8	10
(6,5)	2	3	3
(6,7)	4	4	6

with two alternative MCDM methods, namely the Preference Ranking Organization Method for Enrichment Evaluations (PROMETHEE) and the Copeland score (CS) method.

PROMETHEE [9,10] belongs to a family of outranking methods, or partial aggregation methods (as opposed to complete aggregation methods such as multiattribute utility theory which require the potentially unrealistic synthesis of partial utility functions [17]. The outranking relationship obtained from PROMETHEE, by comparing all of the pairs of alternatives ($n\times(n-1)\times m$ pairwise comparisons) a and b, but instead it establishes if "the alternative a is at least as good as the alternative b" [11]. Multiple criteria are considered when ranking alternatives, and, like TOPSIS, the criteria can be weighted to represent decision maker preferences. In this paper we have used an extension to PROMETHEE referred to as PROMETHEE II to provide a possible total or complete ranking of the alternatives and we used a V-shape/linear preference function. To provide a more accurate comparison with TOPSIS and the CS method, we adopt equal criteria weights. Table 3 provides the net outranking flow $\varnothing$ for each alternative from Fig. 2, which is the difference between the positive outranking flow $\varnothing^+$ and the negative outranking flow $\varnothing^-$, as well as the rank of each alternative where the higher the net outranking flow indicates the more important alternative. More details on the calculation of PROMETHEE, which lie outside of the scope of this paper, can be found in [61].

The CS method is computed based on pairwise comparisons between objects in a set and is defined as the difference between (i) the number of times an alternative a is better (with respect to a particular criterion) than the other alternatives and (ii) the number of times that alternative a is worse (with respect to the same criterion) than the other alternatives, then that difference is summed across all criteria [1]. The CS method is non-parametric in that no weights are assigned to the different criteria. The comparison matrix for all the

Table 4
Comparison matrix and ranks for all alternatives by CS.

Edge	(1,2)	(1,3)	(1,4)	(2,3)	(3,2)	(2,5)	(3,5)	(3,6)	(4,6)	(5,7)	(6,5)	(6,7)	CS	Rank
(1,2)	0	−3	5	−4	−3	0	−4	−6	−6	−1	−2	−4	−28	11
(1,3)	3	0	5	−4	−2	0	−1	−4	−4	0	−2	−4	−13	9
(1,4)	−5	−5	0	−4	−5	−5	−6	−6	−6	−5	−4	−6	−57	12
(2,3)	4	4	4	0	4	4	4	4	2	4	4	4	42	1
(2,5)	0	0	5	−4	0	0	−3	−2	−4	0	−4	0	−12	8
(3,4)	3	2	5	−4	0	0	−2	−1	−4	2	−2	−1	−2	7
(3,5)	4	1	6	−4	2	3	0	0	−4	1	−2	1	8	5
(3,6)	6	4	6	−4	1	2	0	0	−6	4	−1	0	12	4
(4,6)	6	4	6	−2	4	4	4	6	0	4	0	3	39	2
(5,7)	1	0	5	−4	−2	0	−1	−4	−4	0	−2	−3	−14	10
(6,5)	2	2	4	−4	2	4	2	1	0	2	0	2	17	3
(6,7)	4	4	6	−4	1	0	−1	0	−3	3	−2	0	8	6

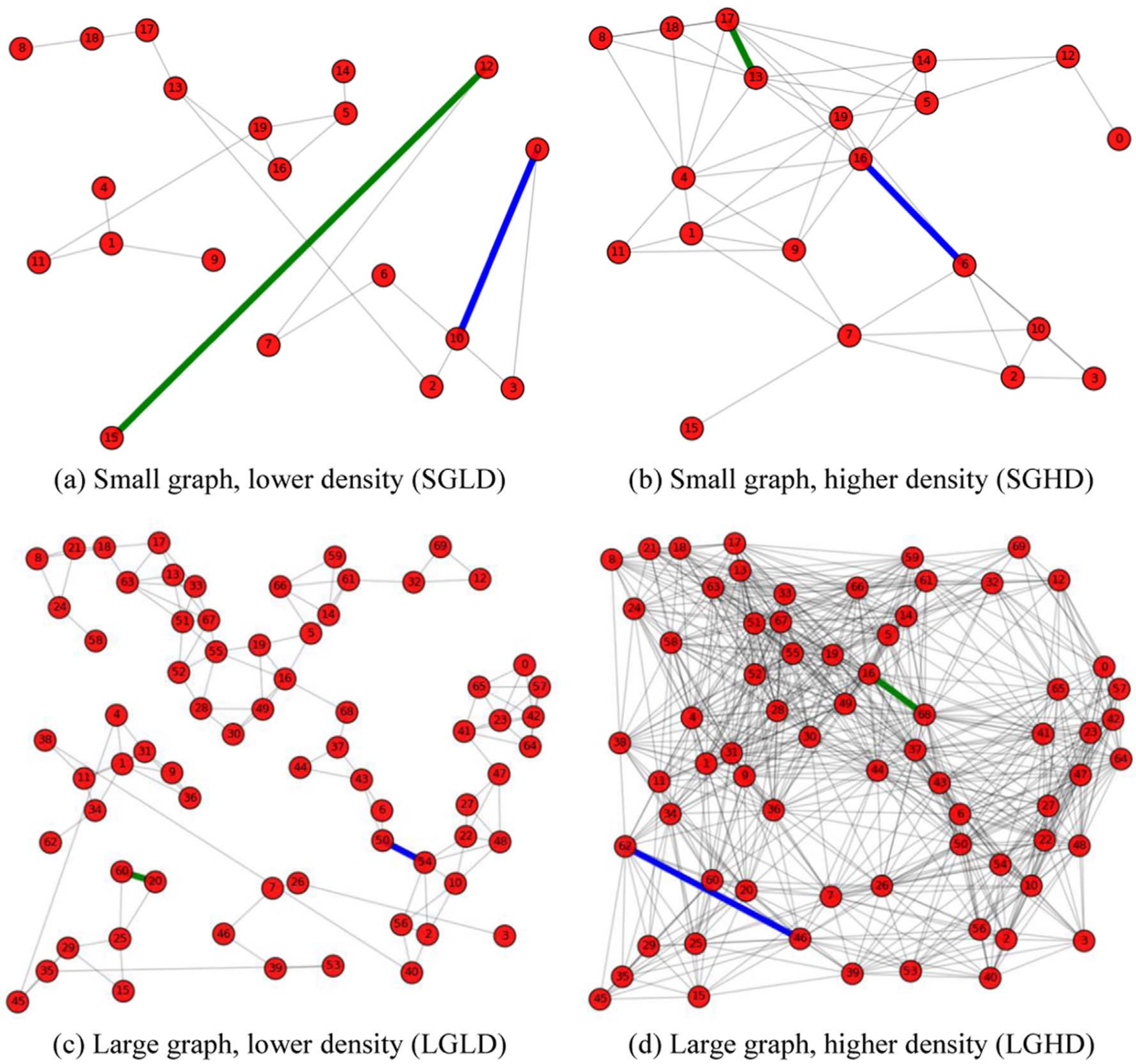

Fig. 4. Four different networks with different densities.

alternatives in Fig. 2 along with their ranks according to their CS are shown in Table 4.

The ranks for all the alternative in the example in Fig. 2 by the proposed approach, TOPSIS, as well PROMETHEE and the CS method, are shown in Table 5. As previously mentioned, equal weights were assigned to the criteria in TOPSIS and PROMETHEE to offer a more direct comparison with the CS method, which does not take into account weights. The ranks in Table 5 shows some agreement between the TOPSIS, PROMETHEE, and CS (e.g., the most important alternative), though many of the rankings are similar (e.g., link (3,4) is considered to be the eighth most important link by two methods and seventh by the other). The rankings for the three selected MCDM methods were compared using the Spearman correlation coefficient (SCC), where an SCC=1 indicates a perfect global association between the rankings. In the example, we obtain SCC(TOPSIS, PROMETHEE) =0.97, SCC(TOPSIS, CS)=0.95, and SCC(PROMETHEE, CS)=0.96, suggesting a very good association among each of the methods when selected pairwise. Further, the W-Kendall coefficient of concordance [34] is 0.97 ($p < 0.0001$), suggesting that there is a very high and significant agreement among the selected MCDM methods.

While this is but one example, it illustrates the similarity of the approaches when no weighting is considered. As we want the ability to give more and less importance to different percentiles, the CS method is not preferred. And between the two methods that allow for percentile weighting, TOPSIS has a much simpler calculation. Other advantages of TOPSIS relative to other techniques include [31,62]: (i) it is easy to use with a sound logic that represents the rationale of human choice, (ii) it gives a scalar value that accounts not only for the best alternative but for the worst too simultaneously, and (iii) it has the ability to measure the relative performance rating for each alternative in a simple mathematical form. As such, we adopt TOPSIS to determine the importance of links with the approach discussed in Fig. 1, and use TOPSIS in the larger illustrative examples that follow.

4. Illustrative examples

In this section, we illustrate our proposed approach with different networks of different sizes and different densities, i.e., with different ration among edges and nodes.

4.1. Data

In this study, we have used the network design instances considered by [41] which are generated from a random geometric graph structure with bi-directional and capacitated edges. A bi-directional edge is composed of two symmetric directed arcs. Capacities are randomly assigned to each edge according to a continuous uniform distribution with the parameters [100, 1000]. Accordingly, four different networks are generated as shown in.

Fig. 4 (a) a small graph with low density (SGLD) instance which has

Table 6
Top 10 ranked edges by each IM for SGLD, SGHD, LGLD, and LGHD networks.

Rank	MFC	MCC	EFC	FCR	WFCR	DI
(a) Small graph, lower density (SGLD)						
1	(2,13)	(0,10)	(13,16)	(0,10)	(0,10)	(0,10)
2	(13,16)	(7,12)	(2,13)	(16,19)	(11,19)	(0,3)
3	(2,10)	(11,19)	(2,10)	(0,3)	(7,12)	(7,12)
4	(0,10)	(8,18)	(0,10)	(11,19)	(5,16)	(11,19)
5	(5,16)	(5,14)	(0,3)	(13,16)	(16,19)	(8,18)
6	(0,3)	(16,19)	(11,19)	(5,16)	(17,18)	(13,16)
7	(16,19)	(17,18)	(3,6)	(7,12)	(2,10)	(5,14)
8	(5,19)	(5,16)	(13,17)	(2,10)	(13,16)	(13,17)
9	(3,6)	(1,4)	(6,7)	(2,13)	(8,18)	(1,4)
10	(11,19)	(2,10)	(1,11)	(13,17)	(1,4)	(5,16)
(b) Small graph, higher density (SGHD)						
1	(6,16)	(7,9)	(6,16)	(7,9)	(6,16)	(6,16)
2	(7,9)	(1,7)	(6,7)	(6,19)	(1,7)	(0,12)
3	(1,7)	(6,19)	(5,12)	(1,7)	(7,9)	(7,15)
4	(6,19)	(6,16)	(5,16)	(6,16)	(6,19)	(1,7)
5	(9,16)	(5,12)	(0,12)	(9,19)	(5,12)	(7,9)
6	(1,16)	(8,18)	(7,15)	(1,16)	(8,18)	(6,19)
7	(1,19)	(0,12)	(1,7)	(4,18)	(1,11)	(3,6)
8	(6,7)	(7,15)	(12,14)	(9,16)	(1,19)	(12,14)
9	(5,16)	(9,11)	(2,6)	(17,18)	(8,17)	(5,12)
10	(9,19)	(8,17)	(1,16)	(4,17)	(14,19)	(8,18)
(c) Large graph, lower density (LGLD)						
1	(50,54)	(4,45)	(50,54)	(50,54)	(50,54)	(4,45)
2	(37,68)	(50,54)	(16,68)	(4,45)	(4,45)	(50,54)
3	(16,68)	(41,47)	(37,68)	(37,68)	(37,68)	(37,68)
4	(37,43)	(37,68)	(38,40)	(11,31)	(41,47)	(41,47)
5	(38,40)	(5,19)	(37,43)	(1,11)	(5,16)	(5,16)
6	(43,50)	(5,16)	(43,50)	(41,47)	(11,34)	(24,58)
7	(11,38)	(11,31)	(11,38)	(43,50)	(16,68)	(32,69)
8	(4,45)	(24,58)	(4,45)	(16,68)	(11,31)	(38,40)
9	(11,31)	(1,11)	(40,56)	(35,45)	(18,63)	(32,61)
10	(1,11)	(11,34)	(2,40)	(40,56)	(38,40)	(16,68)
(d) Large graph, higher density (LGHD)						
1	(7,37)	(29,45)	(46,56)	(7,37)	(46,62)	(15,45)
2	(7,36)	(35,45)	(20,53)	(9,46)	(40,46)	(45,46)
3	(9,46)	(29,35)	(40,46)	(4,60)	(46,56)	(25,35)
4	(4,60)	(15,45)	(46,62)	(34,46)	(20,53)	(25,29)
5	(1,55)	(15,29)	(20,36)	(46,50)	(15,39)	(25,45)
6	(2,7)	(15,35)	(43,55)	(2,6)	(23,32)	(20,45)
7	(46,50)	(0,69)	(34,52)	(43,49)	(2,7)	(15,25)
8	(46,62)	(45,62)	(16,41)	(7,36)	(15,46)	(0,12)
9	(34,52)	(29,62)	(47,56)	(46,53)	(34,62)	(15,29)
10	(46,56)	(15,39)	(14,43)	(0,61)	(35,62)	(29,45)

Table 7
Top 10 ranked edges by the proposed approach for SGLD, SGHD, LGLD, and LGHD networks considering equal weights for each IM.

Rank	SGLD	SGHD	LGLD	LGHD
1	(0,10)	(6,16)	(50,54)	(46,62)
2	(11,19)	(1,7)	(4,45)	(15,39)
3	(7,12)	(7,9)	(37,68)	(15,45)
4	(0,3)	(6,19)	(41,47)	(40,46)
5	(16,19)	(5,12)	(16,68)	(45,46)
6	(13,16)	(0,12)	(43,50)	(20,53)
7	(2,10)	(7,15)	(38,40)	(46,56)
8	(2,13)	(12,14)	(37,43)	(35,62)
9	(5,16)	(6,7)	(11,31)	(20,45)
10	(8,18)	(8,18)	(11,38)	(15,46)

Table 8
The most and least important edge for SGLD, SGHD, LGLD, and LGHD networks.

Network	**SGLD**	**SGHD**	**LGLD**	**LGHD**
Most important edge	(0,10)	(6,16)	(50,54)	(46,62)
Least important edge	(12,15)	(13,17)	(20,60)	(16,68)

20 bi-directional edges, (b) a small graph with high density (SGHD) which has 53 bi-directional edges, (c) a large graph with low density (LGLD) which has 128 bi-directional edges, and (d) a large graph with high density (LGHD) which has 791 bi-directional edges.

4.2. Alternative (edges) ranks

We first obtain the score of each edge by each IM and rank them accordingly where the largest score is the most important edge. So, Table 6 shows the top 10 ranked edges by each IM for SGLD, SGHD, LGLD, and LGHD networks, respectively. As in the previous example, no edge in the network analyzed is considered as the dominant edge. We then follow the procedure of the proposed approach as shown in Fig. 1 to obtain a unique rank for the network edges according to their importance that assesses different aspects of the network performance and lead to improve the network desired function (e.g., flow). The top ten edges for each network according to the proposed approach considering equal weights for each IM are shown in Table 7.

Table 8 shows the most important edge and the least important edge according to the proposed approach for SGLD, SGHD, LGLD, and LGHD networks, respectively. They are also shown in Fig. 4 where the most important edges are in blue color and the least important edges are in green color.

4.3. Effects of weights

In general, a weight could be assigned to each IMs to emphasize its value. A possible approach could be simply to choose reasonable weights for the selected IMs (such that $\sum_{k=1}^{m} w_k=1$). Tervonen and Lahdelma [59] present several approaches that account for different decision maker preference information such as absent preference, interval constraints for weights or complete ranking of the criteria.

To illustrate how the weighting of the IMs could affect the ranking of the edges, we have applied the proposed approach to the four networks above with a large number of different sets of weights (1000 generated sets of weights simulated from a uniform distribution between (0,1), i.e., without any preference information provided by the decision maker) by using the procedure in Tervonen and Lahdelma [59]) to find the probability of the occurrence of each edge in each ranking position [49]. This uncertainty propagation analysis (i.e., the effects of the uncertainty of the weights) is similar to the SMAA-PROMETHEE approach of [15].

Fig. 5 shows the heat maps for the top ten ranking positions. Each heat map displays the top ten highly probable ranked edges where the darker intersection represents the higher probability of that edge being ranked in that position. For example, edges (0,10), (6,16) and (50,54) are always ranked as the most important edges for SGLD, SGHD, and LGLD networks, respectively, with the consideration of 1000 randomly generated sets of weights as shown in Fig. 5(a)–(c). Similarly, edge (46,62) has more probability than other edges to be ranked as the most important alternative for LGHD network, see Fig. 5(d). However, for this network, and depending on the weights selected by the decision maker, edge (15,45) has a high probability of be the most important edge.

4.3.1. Effects of selected weighting schemes

TOPSIS can incorporate relative weights of criterion importance [42]. Hence, it is attractive since the subjective input that is needed from decision makers is limited only to the weights of criteria. Naturally, depending on the weights source selected, the edges could be ranked in different positions. In this section, we consider three different weighting schemes: (i) equal weights, (ii) static weights, such as those obtained using an elicitation technique (e.g., analytic hierarchy process (AHP) [53], and (iii) random weights based on uniform distribution (i.e., U(0,1) as per Section 4.3). Accordingly, we have obtained the ranks of the alternatives for the four network densities

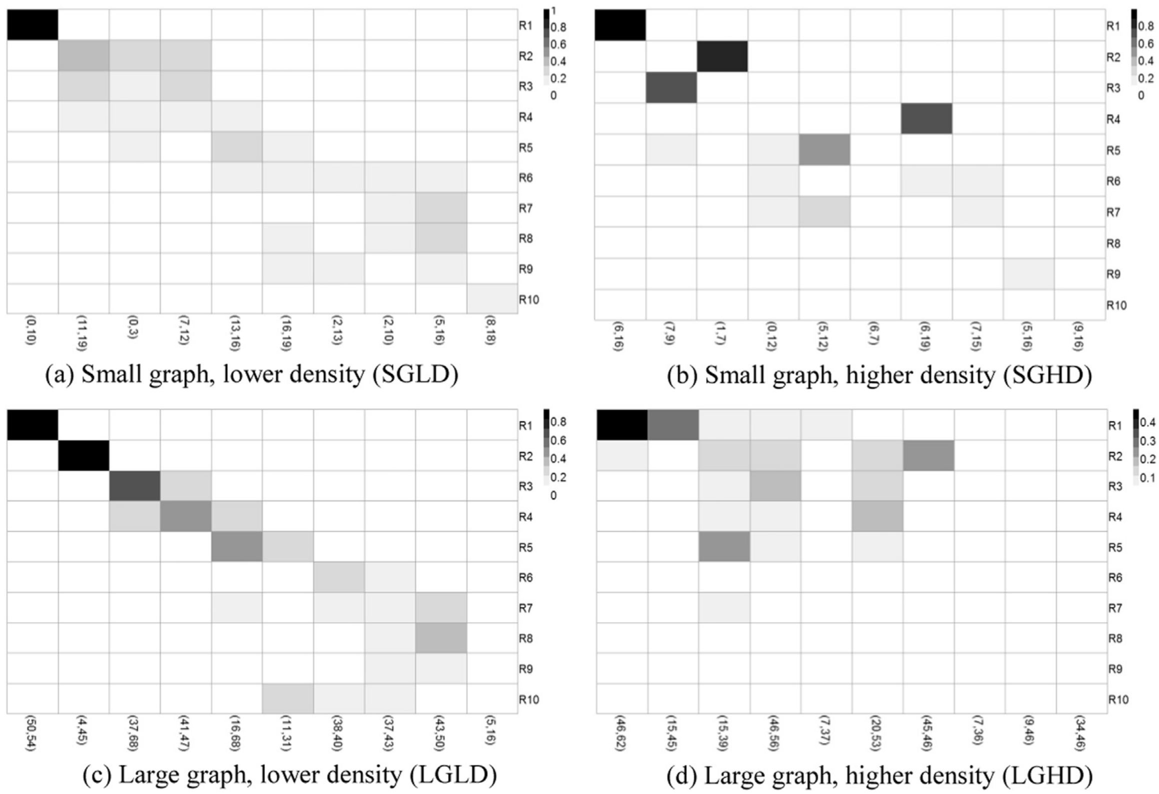

Fig. 5. Heat maps showing the probability of the ranking (vertical axis) of the top ten alternatives (horizontal axis).

(SGLD, SGHD, LGLD, LGHD) using these three different weight sources. The top ten important edges for each network according to the ranks obtained by TOPSIS based on the edges importance and considering different sources of weights are found in Table 9. In addition, the bottom ten alternatives for each network are presented in Table 10 as well.

Though different weight sources are considered which produce different ranks for the alternatives by TOPSIS, there is an agreement on the most important edge(s) obtained by TOPSIS as shown in Table 9. For instance, edges (0,10), (6,16), (50,54), and (46,62) are always the most critical edges for SGLD, SGHD, LGLD, and LGHD densities, respectively, regardless of the source of weights. Likewise, there is an agreement on the worst alternative at least for two networks: edges (12,15) and (20,60) are considered the least critical for SGLD and LGLD networks, respectively, according to the ranks obtained by TOPSIS with different sets of weights for the criteria, as shown in Table 10(a) and (c). Also, edge (16,68) is considered the least critical for LGHD network according to two different weighting schemes as shown in Table 10(d).

5. Concluding remarks

Analyzing network vulnerability is a key element of network preparedness planning in advance of disruptive events that might impact the performance of the network. Hence, many importance measures (IMs) have been proposed to identify the important components in a network with respect to vulnerability. Such IMs produce valuable information for decision makers, including highlighting the most important network components according to a specific performance function. However, different IMs based on different definitions could lead to different rankings of network components according to their importance. Hence, a challenge that is faced by decision makers is to combine such information from different IMs and generate a unique rank of network components [47]. To address this challenge, we propose a new approach in this paper to identify the most important network components through a unique rank based on multiple IMs with different perspectives using a multi-criteria decision analysis tool, TOPSIS, which can account for decision maker preferences on the importance of different importance perspectives. The multiple flow-based IMs provided by [41] are considered as the multiple criteria with which to compare the importance of network edges. TOPSIS is then utilized to aggregate these different IMs to find the most important edges of the network.

The proposed approach is illustrated by assessing the components (edges) of four different networks with different densities under six different flow-based IMs. A unique rank of edges for each network is obtained accordingly by the proposed approach instead of multiple ranking derived by different IMs with different perspectives. Using the proposed approach, the edges of each network are ranked based on their similarities to the positive ideal solution of each network where the larger similarity indicates the more important edge. Moreover, we have applied the proposed approach to the four networks with a large number of different generated sets of weights that are simulated from a uniform distribution between (0,1) in order to find the probability of the occurrence of each edge in each ranking position. The first position was dominated by a single edge in the small graph with low density (SGLD), the small graph with high density (SGHD) and the large graph with low density (LGLD) networks while it is not the case for the large graph with high density (LGHD) network which shows that the larger size the network is the less domination will be by any edge for any ranking position. Furthermore, since the only subjective input that is needed from decision makers is the weights of criteria, we have obtained the ranks of the alternatives for the four networks by the proposed approach considering three different weight sources (i.e., equal weights, analytic hierarchy process (AHP), and random weights based on U(0,1)). Though different weights sources are considered which produce different ranks for the alternatives, there was an agreement on the most preferred alternative(s) obtained by the proposed approach for all the four networks but not for the least preferred alternative(s) for SGHD and LGHD. Despite this agreement

Table 9
Top 10 ranked edges by the proposed approach with different weights criteria.

Equal weight	Static	Random weight
(a) Small graph, lower density (SGLD)		
(0,10)	**(0,10)**	**(0,10)**
(11,19)	**(11,19)**	**(11,19)**
(7,12)	(7,12)	(0,3)
(0,3)	(16,19)	(7,12)
(16,19)	(5,16)	(13,16)
(13,16)	(0,3)	(2,10)
(2,10)	(13,16)	(2,13)
(2,13)	(2,10)	(5,16)
(5,16)	(17,18)	(16,19)
(8,18)	(2,13)	(3,6)
(b) Small graph, higher density (SGHD)		
(6,16)	**(6,16)**	**(6,16)**
(1,7)	**(1,7)**	**(1,7)**
(7,9)	**(7,9)**	**(7,9)**
(6,19)	**(6,19)**	**(6,19)**
(5,12)	**(5,12)**	**(5,12)**
(0,12)	(8,18)	(7,15)
(7,15)	(1,19)	(0,12)
(12,14)	(1,11)	(6,7)
(6,7)	(8,17)	(12,14)
(8,18)	(14,19)	(1,19)
(c) Large graph, lower density (LGLD)		
(50,54)	**(50,54)**	**(50,54)**
(4,45)	**(4,45)**	**(4,45)**
(37,68)	**(37,68)**	**(37,68)**
(41,47)	**(41,47)**	**(41,47)**
(16,68)	(11,31)	(16,68)
(43,50)	(1,11)	(38,40)
(38,40)	(16,68)	(37,43)
(37,43)	(43,50)	(43,50)
(11,31)	(5,16)	(11,38)
(11,38)	(37,43)	(40,56)
(d) Large graph, higher density (LGHD)		
(46,62)	**(46,62)**	**(46,62)**
(15,39)	(40,46)	(15,39)
(15,45)	(46,56)	(40,46)
(40,46)	(20,53)	(20,53)
(45,46)	(15,39)	(46,56)
(20,53)	(23,32)	(15,45)
(46,56)	(2,7)	(45,46)
(35,62)	(7,34)	(35,62)
(20,45)	(34,62)	(15,46)
(15,46)	(15,46)	(23,32)

Table 10
Bottom 10 ranked edges by the proposed approach with different weights criteria.

Equal weight	Static	Random weight
(a) Small graph, lower density (SGLD)		
(13,17)	(8,18)	(13,17)
(3,6)	(13,17)	(17,18)
(17,18)	(5,14)	(5,19)
(5,14)	(1,4)	(1,11)
(1,11)	(1,11)	(6,7)
(1,4)	(6,7)	(8,18)
(6,7)	(5,19)	(5,14)
(5,19)	(3,6)	(1,4)
(1,9)	**(1,9)**	**(1,9)**
(12,15)	**(12,15)**	**(12,15)**
(b) Small graph, higher density (SGHD)		
(5,13)	(13,16)	(5,13)
(8,13)	(3,6)	(17,18)
(2,7)	(2,7)	(4,13)
(4,13)	(3,10)	(2,10)
(16,19)	(13,14)	(16,19)
(13,14)	(2,6)	(6,10)
(2,3)	(6,10)	(8,13)
(6,10)	(13,17)	(2,3)
(2,10)	(2,3)	(13,17)
(13,17)	(2,10)	(13,14)
(c) Large graph, lower density (LGLD)		
(15,25)	(0,65)	(15,25)
(57,65)	(23,57)	(14,66)
(0,57)	(14,66)	(14,61)
(14,61)	(57,65)	(23,42)
(23,64)	(0,57)	(23,64)
(14,66)	(23,42)	(42,65)
(23,42)	(23,64)	(42,64)
(42,65)	(42,65)	(0,57)
(23,65)	**(23,65)**	**(23,65)**
(20,60)	**(20,60)**	**(20,60)**
(d) Large graph, higher density (LGHD)		
(28,49)	(28,67)	(19,51)
(30,31)	(16,67)	(17,63)
(44,49)	(44,68)	(30,31)
(36,44)	(19,51)	(28,31)
(16,67)	(43,68)	(16,67)
(19,55)	(19,55)	(1,52)
(19,51)	(30,36)	(28,55)
(30,36)	(49,51)	(19,55)
(49,51)	(44,49)	(16,68)
(16,68)	(16,68)	(28,67)

which could be resulted from the size, density, and setup (connections) of the considered problems, different weights provided by decision makers for the IMs could result in different ranks for network components especially for larger size and higher dense networks.

The proposed approach could be used in the case of a general multi criteria problem, where the identification of importance and the ranking are desired for a set of alternatives (e.g., network components) through a set of criteria (e.g., edge importance measures).

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